



*Badan Pusat Statistik
Provinsi Sumatera Utara*

MODEL PERSAMAAN SIMULTAN DATA PANEL UNTUK *SMALL AREA ESTIMATION*

Reny Ari Noviyanti
NIP 198011122002122001
[Email : reny.ari@bps.go.id](mailto:reny.ari@bps.go.id)



23 April 2025



Aula Lt 5 BPS Prov Sumatera Utara

Presented at:

WEGO

(Wadah Edukasi dan Gagasan Optimal)

Small Area Estimation :
Menduga Akurat dengan Lebih Hemat



KNOWLEDGE SHARING

Presented by:
Reny Ari Noviyanti

PENDAHULUAN

MODEL SERY

SIMULASI MODEL

PENERAPAN MODEL

KESIMPULAN

SARAN

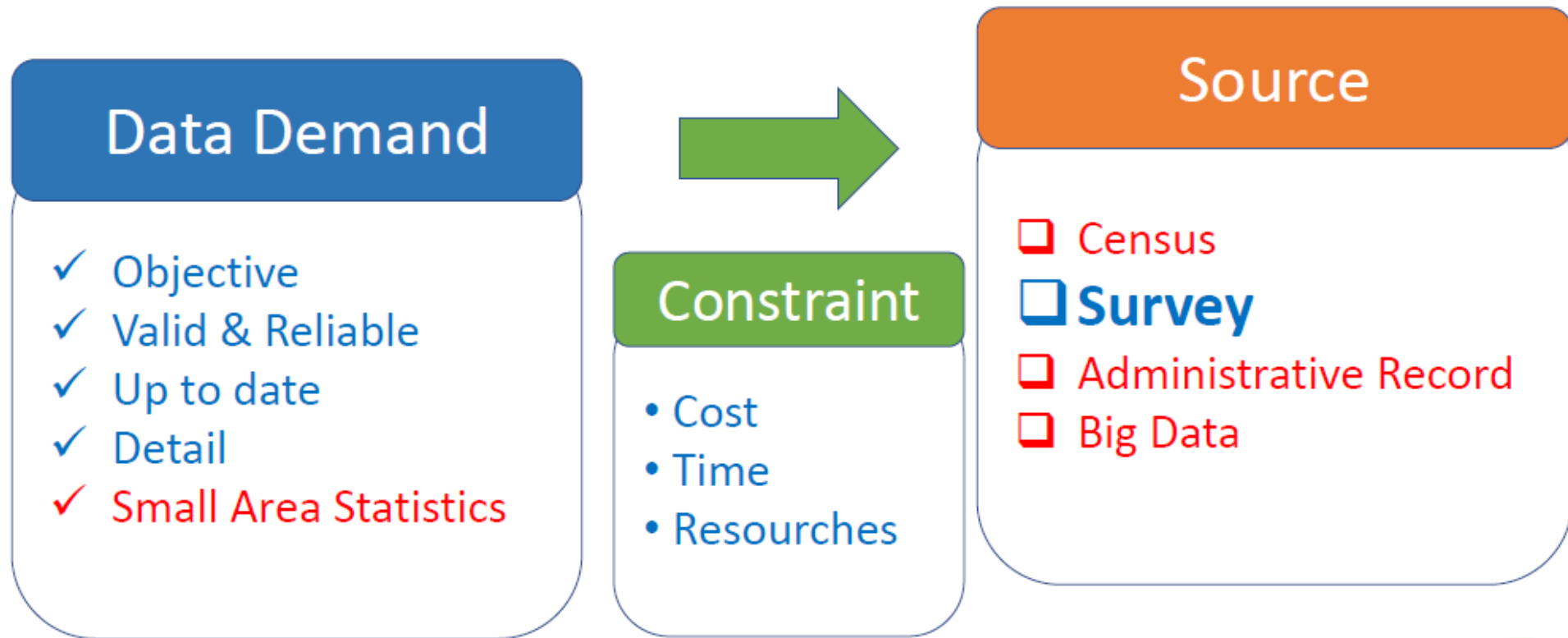


PENDAHULUAN



Pendahuluan

Kebutuhan data



Pendahuluan

Survei



- Survei dirancang untuk estimasi parameter sampai level tertentu, baik level wilayah administratif maupun level kelompok tertentu.
- Estimasi parameter menggunakan formula dari desain survei (dengan kalibrasi penimbang/*weight*) disebut *direct estimation*.
- *Direct estimation* pada level estimasi sesuai desain survei dipercaya mampu menghasilkan estimator parameter populasi yang baik.
- Salah satu kriteria yang digunakan untuk menilai kebaikan hasil estimasi adalah *Relative Standard Error* (RSE)

Pendahuluan

Area Luas VS Area Kecil

Area Luas

- ✓ Wilayah dimana survei dirancang level estimasinya
- ✓ Kecukupan ukuran sampel terpenuhi
- ✓ *Direct estimation* bisa dipercaya

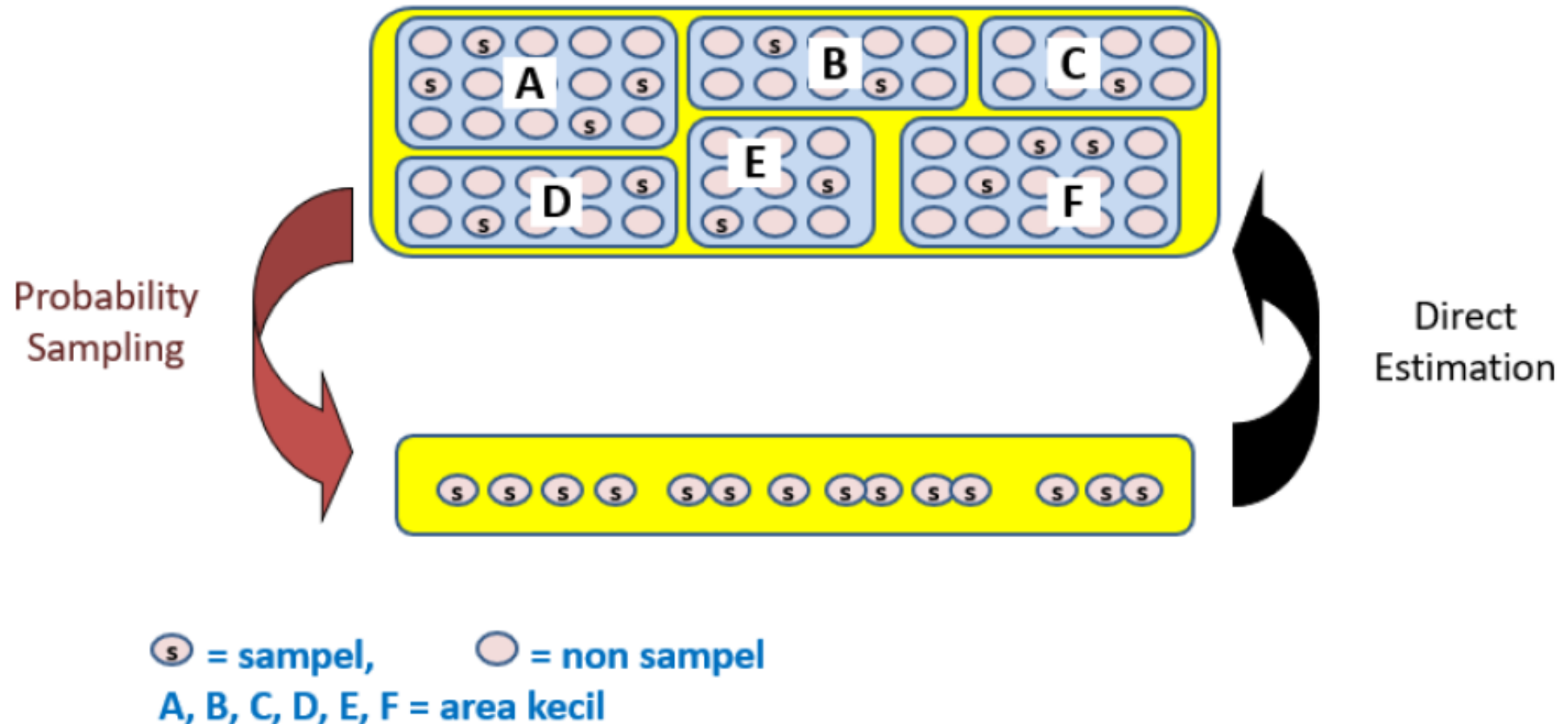


Area Kecil

- ✓ Wilayah dimana survei **tidak** dirancang level estimasinya
- ✓ Kecukupan ukuran sampel **tidak** terpenuhi
- ✓ *Direct estimation* **tidak** bisa dipercaya

Pendahuluan

Ilustrasi Area Kecil

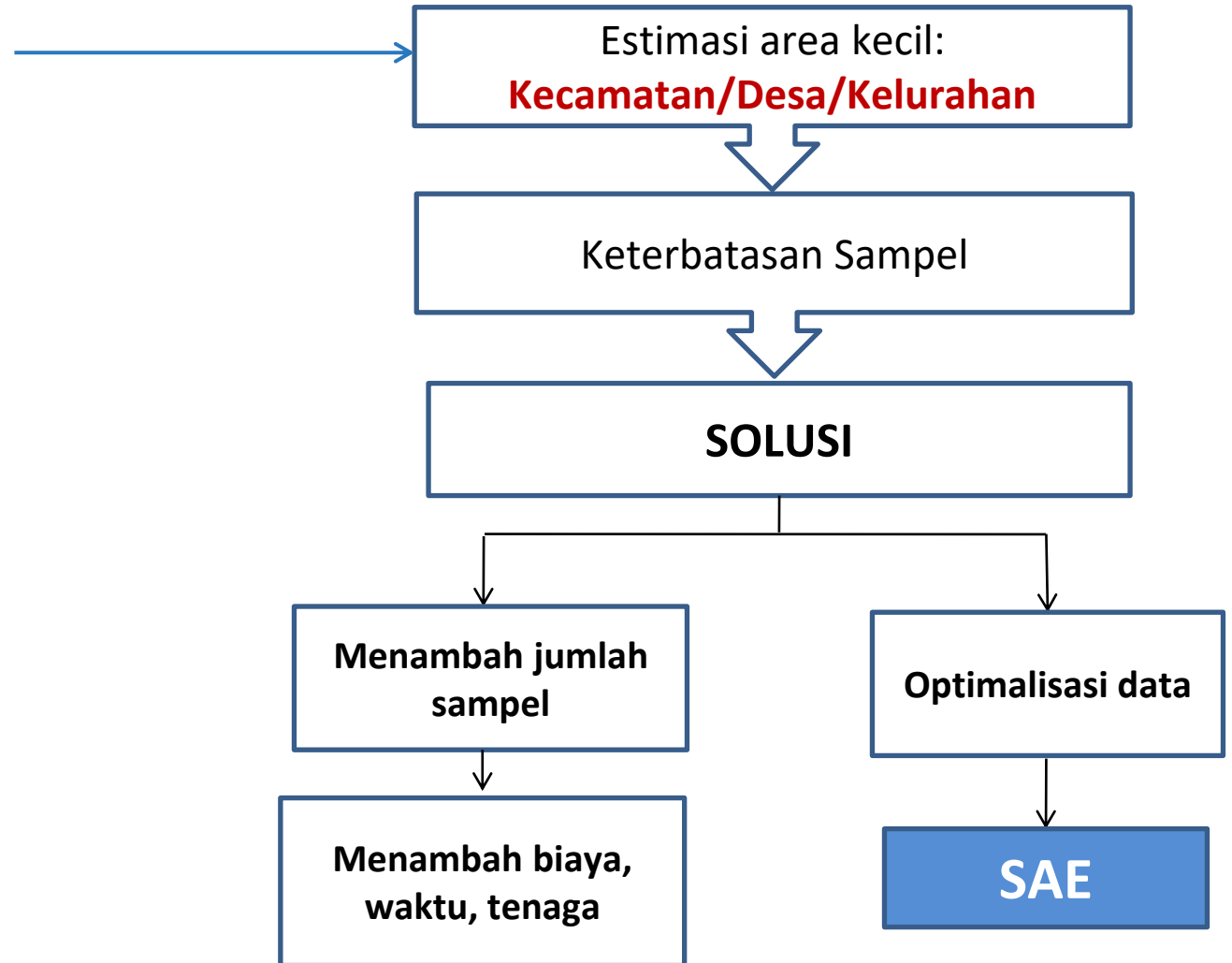


Pendahuluan



Survei rutin:
Susenas
(Maret dan September
setiap tahun)

Estimasi:
Nasional,
Provinsi,
Kabupaten/Kota



Pendahuluan

Mengapa perlu SAE?

01

Era otonomi dan desentralisasi semakin menuntut ketersediaan data di level area kecil

02

Survei umumnya mempunyai ukuran sampel terbatas dan hanya mampu dilakukan estimasi sampai level tertentu.

03

Direct estimation pada area kecil (ukuran sampel tidak cukup) sehingga menghasilkan **standard error** (kesalahan baku) yang besar.

04

Menambah jumlah sampel berakibat meningkatnya biaya, waktu, tenaga, *non-sampling error*, dsb

Pendahuluan

Metode SAE

1

Metode SAE dikembangkan untuk mengatasi permasalahan **kurangnya ukuran sampel** di level area kecil.

2

Metode yang **mengkombinasikan** data survei yang tersedia dengan data pendukung lain.

3

SAE dibutuhkan untuk meningkatkan **efektivitas** ukuran sampel dan meningkatkan **efisiensi** estimasi parameter.



Pendahuluan

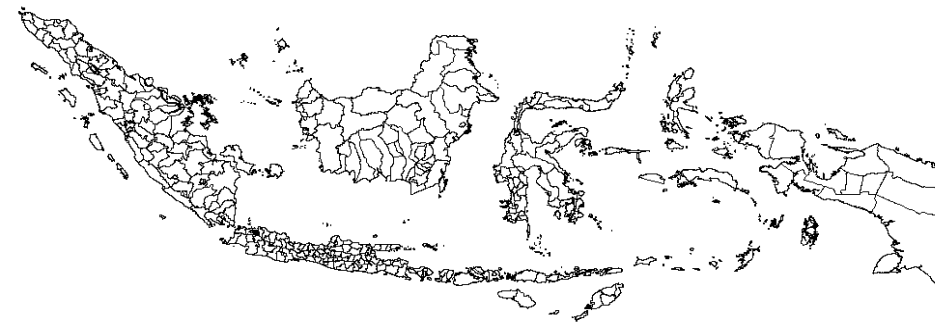
Kata kunci dalam pendugaan area kecil (*small area estimation*, SAE):

- “*small in sample*” → ukuran contoh yang kecil, **bukan** “*area* = wilayah” yang kecil.
- “*area*” tidak selalu identik dengan wilayah administratif/geografis, lebih umum merupakan *sub-domain*/himpunan dari populasi.

Variabel penyerta(*auxiliary variable*)

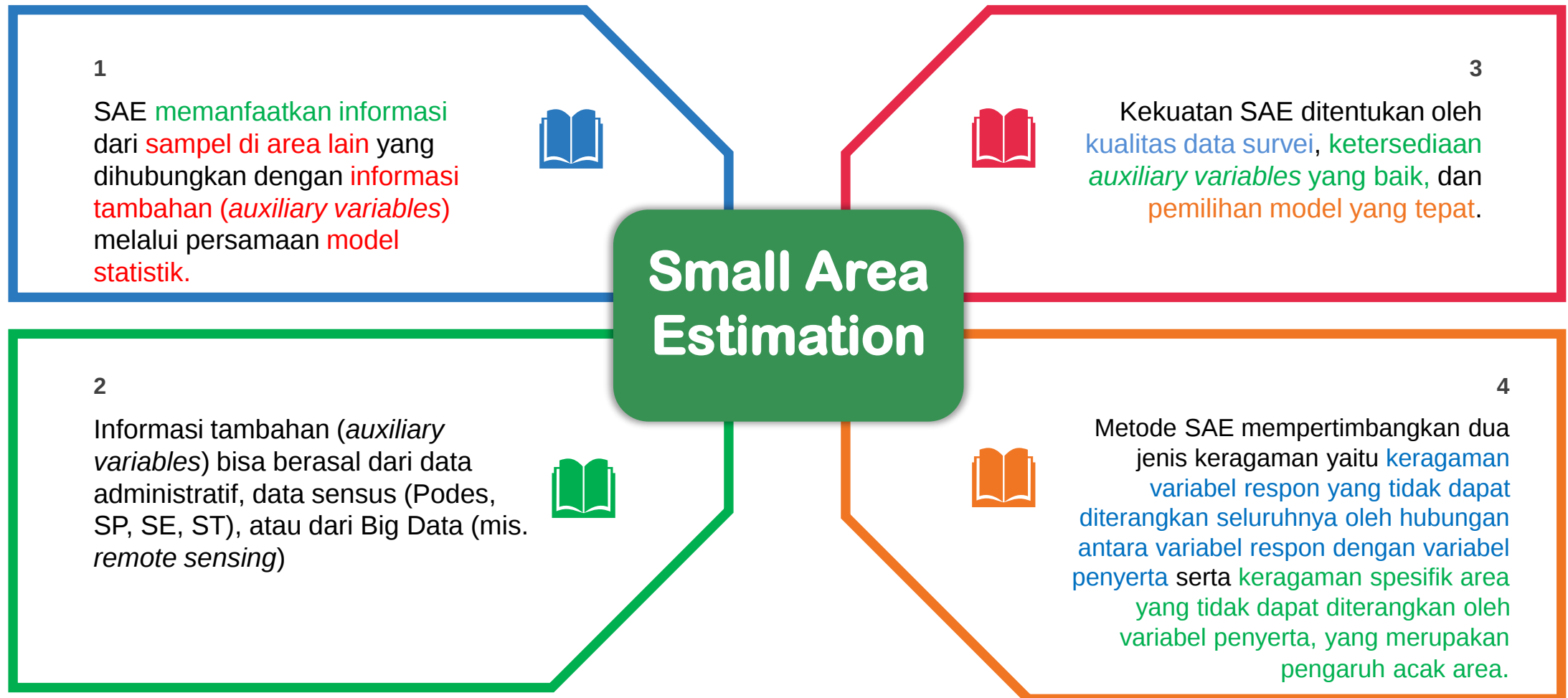
→ Menambahkan informasi

“Upaya untuk meningkatkan efektifitas ukuran contoh”



Pendahuluan

Bagaimana SAE bekerja?



Pendahuluan

Level Estimasi SAE

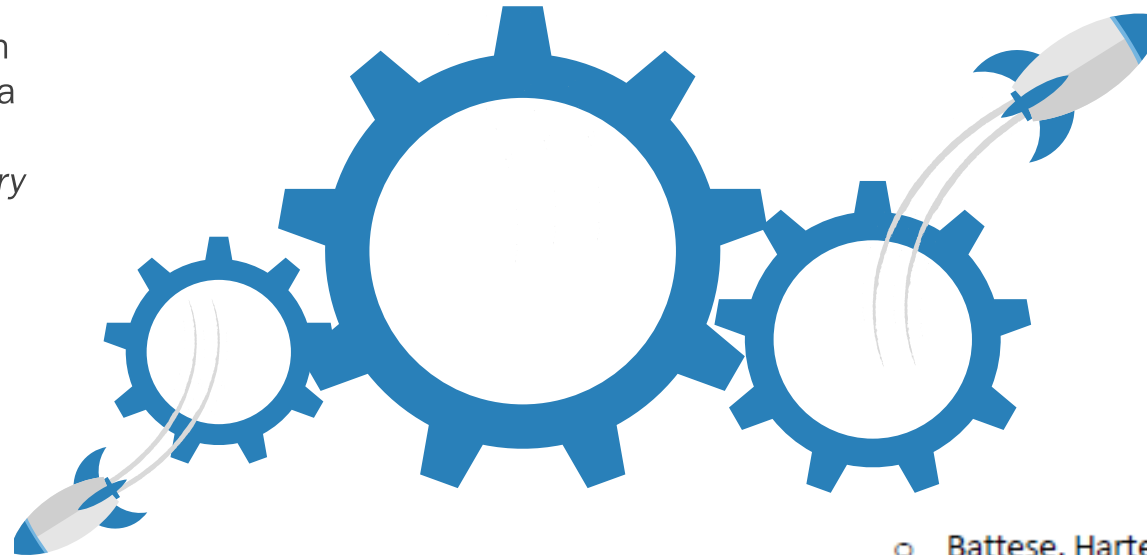
Berdasarkan ketersediaan *auxiliary variables*, model SAE terdiri dari:

MODEL LEVEL AREA

- Model yang menghubungkan estimator langsung dari area kecil dengan data pendukung/kovariat (*auxiliary variables*) dari area yang bersangkutan.

- Fay dan Herriot (1979).
- Peubah penyejajanya hanya tersedia pada level area.

$$\text{Model : } \left. \begin{array}{l} \tilde{y}_i = \bar{y}_i + e_i \\ \bar{y}_i = x_i^T \beta + v_i \end{array} \right\} \tilde{y}_i = x_i^T \beta + v_i + e_i$$



MODEL LEVEL UNIT

Model yang menghubungkan nilai-nilai unit dari variabel yang menjadi perhatian (yang ingin di estimasi agregasinya) dengan kovariat unit-spesifik dari area yang bersangkutan (area yang ingin diestimasi parameternya).

- Battese, Harter and Fuller (1988).
- Peubah penyejajanya tersedia pada level unit.

$$\text{Model : } y_{ij} = x_{ij}^T \beta + v_i + e_{ij}; v_i \sim N(0, \sigma_v^2), e_{ij} \sim N(0, \sigma_e^2)$$

Pendahuluan

Metode estimasi parameter pada model SAE

EBLUP (Empirical Best Linear Unbiased Prediction).

Metode untuk memperkirakan parameter dengan cara meminimumkan Mean Square Error (MSE)



Hierarchical Bayes (HB)

Estimasi didasarkan pada distribusi posterior dimana parameter diestimasi dengan rata-rata posterior dan presisinya diukur dengan varians posteriornya

Empirical Bayes (EB)

Metode estimasi parameter pada SAE yang didasarkan pada metode Bayes dimana inferensia yang diperoleh berdasar pada estimasi distribusi posterior dari variabel yang diamati.

Pendahuluan

Linear Mixed Models (LMM)

Parameter *small area* yang akan diestimasi adalah θ_i :

$$\theta_i = \mathbf{x}_i' \boldsymbol{\beta} + z_i v_i \quad i = 1, 2, \dots, m$$

dengan

$\boldsymbol{\beta} = (\beta_1, \dots, \beta_q)'$ adalah vektor parameter yang *fixed*, berukuran $q \times 1$,

z_i = konstanta positif yang diketahui,

v_i = pengaruh acak *small area*, diasumsikan $v_i \sim iid N(0, \sigma_v^2)$.

Estimasi langsung $\hat{Y}_i$ diketahui dan dinyatakan sebagai:

$$y_i = \hat{\theta}_i = g(\hat{Y}_i) = \theta_i + e_i \quad i = 1, 2, \dots, m$$

dimana e_i adalah *sampling error*, diasumsikan $e_i \sim iid N(0, \sigma_e^2)$, dimana σ_e^2

adalah varian *sampling error* yang diasumsikan diketahui.

Model *Small Area*:

$$\hat{\theta}_i = \mathbf{x}_i^T \hat{\boldsymbol{\beta}} + z_i v_i + e_i$$

Random Effect

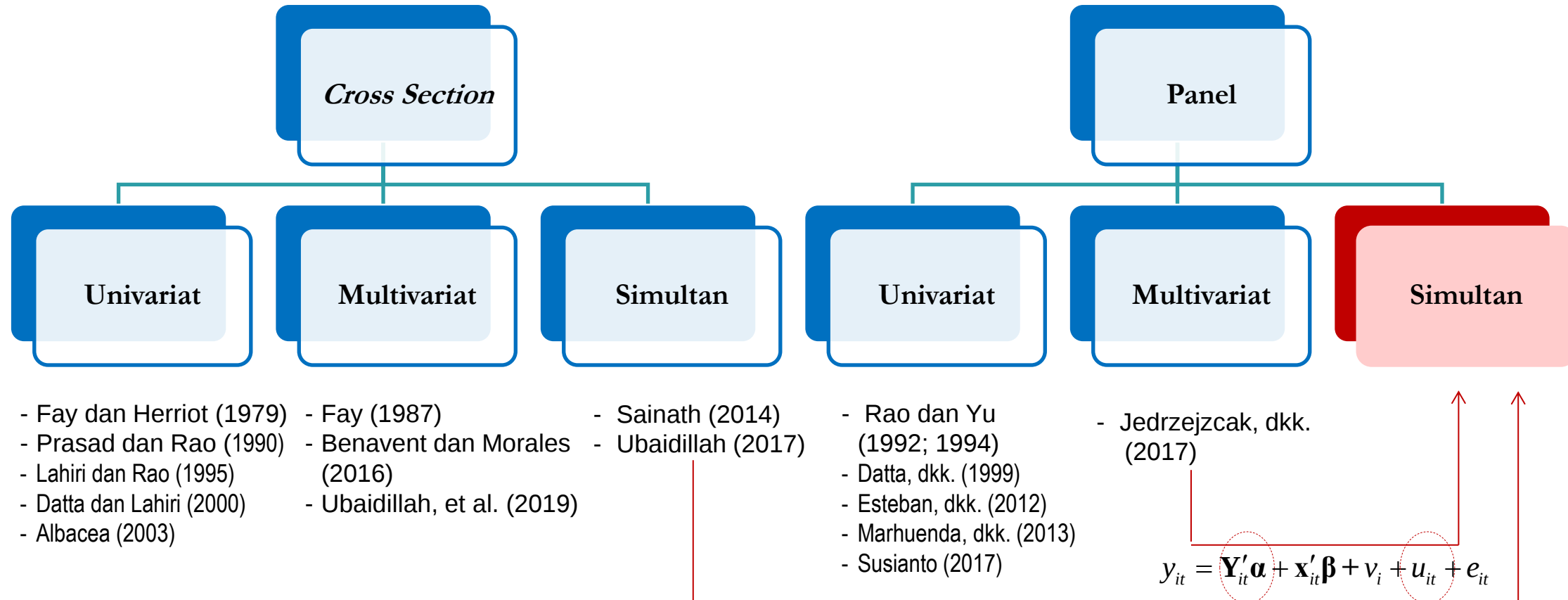
Fixed Effect

Pendahuluan

Small Area Estimation (SAE)

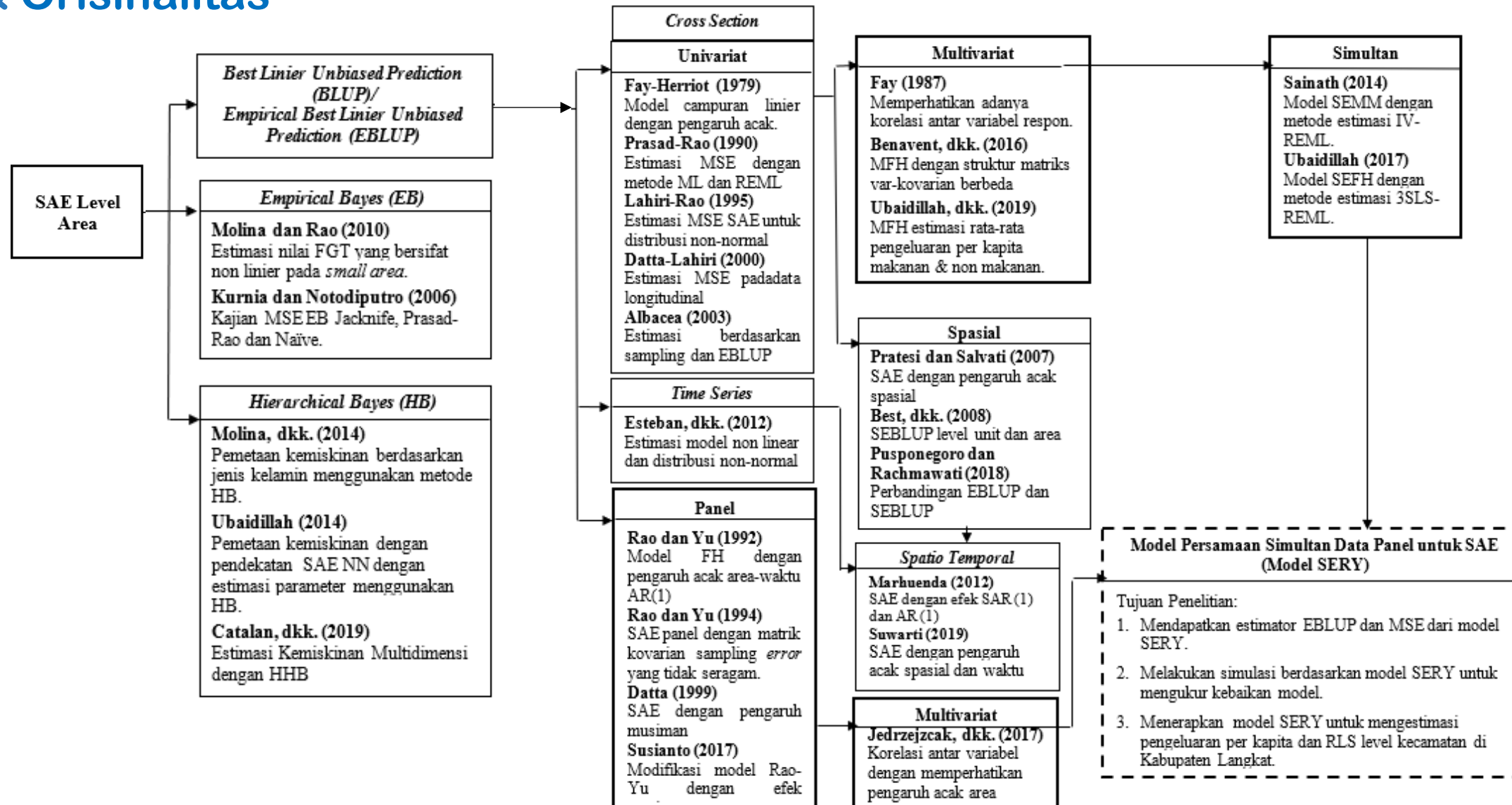
🎯 Model level unit

🎯 Model level area



Pendahuluan

Kontribusi & Orisinalitas Penelitian



ESTIMASI PARAMETER



ESTIMASI PARAMETER

Univariate Fay-Herriot (UFH)

Variabel respon yang diestimasi hanya satu

$$\left. \begin{aligned} \theta_i &= \mathbf{x}_i^T \boldsymbol{\beta} + z_i v_i, \quad i = 1, 2, \dots, m \\ v_i &\sim iid N(0, \sigma_v^2) \\ y_i &= \hat{\theta}_i = g(\hat{Y}_i) = \theta_i + e_i, \quad i = 1, 2, \dots, m \\ e_i &\sim iid N(0, \sigma_e^2) \end{aligned} \right\} \begin{aligned} y_i &= \mathbf{x}_i' \boldsymbol{\beta} + z_i v_i + e_i \\ \mathbf{y} &= \mathbf{X}\boldsymbol{\beta} + \mathbf{Z}\mathbf{v} + \mathbf{e} \end{aligned}$$

Estimator EBLUP

$$\begin{aligned} \mathbf{v} &\stackrel{iid}{\sim} N(0, \mathbf{G}) \text{ dan } \mathbf{e} \stackrel{iid}{\sim} N(0, \mathbf{R}) \\ \mathbf{G} &= \mathbf{I}_m \sigma_v^2 & \mathbf{R} &= \mathbf{I}_m \sigma_e^2 \\ \boldsymbol{\Omega} &= \mathbf{Z}\mathbf{G}\mathbf{Z}^T + \mathbf{R} \end{aligned}$$

$$\begin{aligned} \hat{\boldsymbol{\theta}}^H(\hat{\boldsymbol{\delta}}) &= \mathbf{X}\hat{\boldsymbol{\beta}} + \mathbf{Z}\hat{\mathbf{G}}\mathbf{Z}^T \hat{\boldsymbol{\Omega}}^{-1} (\mathbf{y} - \mathbf{X}\hat{\boldsymbol{\beta}}) \\ \hat{\boldsymbol{\beta}} &= \hat{\boldsymbol{\beta}}(\hat{\boldsymbol{\delta}}) = (\mathbf{X}^T \hat{\boldsymbol{\Omega}}^{-1} \mathbf{X})^{-1} (\mathbf{X}^T \hat{\boldsymbol{\Omega}}^{-1} \mathbf{y}) \end{aligned}$$

Multivariate Fay-Herriot (MFH)

Variabel respon yang diestimasi lebih dari satu yang saling berkorelasi

$$\left. \begin{aligned} \boldsymbol{\theta}_i &= \mathbf{X}_i \boldsymbol{\beta} + \mathbf{v}_i, \quad \mathbf{v}_i \sim N(0, \mathbf{G}_v) \\ \boldsymbol{\theta}_i &= (\theta_{i1}, \dots, \theta_{iD})^T \\ i &= 1, \dots, m \text{ dan } d = (1, \dots, D) \\ \mathbf{v}_i &= (v_{i1}, \dots, v_{iD})^T \quad \mathbf{G}_v = \text{diag}_{1 \leq d \leq D}(\sigma_v^2) \end{aligned} \right\} \begin{aligned} \mathbf{y}_i &= \mathbf{X}_i \boldsymbol{\beta} + \mathbf{v}_i + \mathbf{e}_i \\ \mathbf{y} &= \mathbf{X}\boldsymbol{\beta} + \mathbf{Z}\mathbf{v} + \mathbf{e}, \end{aligned}$$

$$\begin{aligned} \mathbf{y}_i &= \boldsymbol{\theta}_i + \mathbf{e}_i, \quad \mathbf{e}_i \sim N(0, \mathbf{R}_e) \\ \mathbf{y}_i &= (y_{i1}, \dots, y_{iD})^T \end{aligned}$$

$$\begin{aligned} \mathbf{v} &\sim N(0, \mathbf{G}), \quad \mathbf{e} \sim N(0, \mathbf{R}) \\ \mathbf{v} &= \text{col}_{1 < d < D}(\mathbf{v}_d) \text{ dan } \mathbf{e} = \text{col}_{1 < d < D}(\mathbf{e}_d) \end{aligned}$$

Estimator EBLUP

$$\begin{aligned} \hat{\boldsymbol{\theta}} &= \mathbf{X}\hat{\boldsymbol{\beta}} + \mathbf{Z}\hat{\mathbf{G}}\mathbf{Z}^T \hat{\boldsymbol{\Omega}}^{-1} (\mathbf{y} - \mathbf{X}\hat{\boldsymbol{\beta}}) \\ \hat{\boldsymbol{\beta}} &= (\mathbf{X}^T \hat{\boldsymbol{\Omega}}^{-1} \mathbf{X})^{-1} \mathbf{X}^T \hat{\boldsymbol{\Omega}}^{-1} \mathbf{y} \end{aligned}$$

ESTIMASI PARAMETER

Univariate Rao-Yu

Pengembangan dari model Fay-Herriot, dengan menambahkan komponen pengaruh acak area-waktu mengikuti proses autoregresif ordo pertama sehingga model bisa digunakan untuk analisis data panel.

$$\left. \begin{aligned} \bar{y}_{it} &= \theta_{it} + e_{it} \\ \theta_{it} &= \mathbf{x}_{it}^T \boldsymbol{\beta} + v_i + u_{it} \\ u_{it} &= \rho u_{i,t-1} + \varepsilon_{it}, \quad |\rho| < 1 \\ i &= 1, 2, \dots, m \\ t &= 1, 2, \dots, T \end{aligned} \right\} \begin{aligned} \bar{y}_{it} &= \mathbf{x}_{it}^T \boldsymbol{\beta} + v_i + u_{it} + e_{it}, \\ u_{it} &= \rho u_{i,t-1} + \varepsilon_{it}, \quad |\rho| < 1, \\ i &= 1, \dots, m, \quad t = 1, \dots, T \end{aligned}$$

Estimator EBLUP

$$\begin{aligned} \hat{\theta}_{it_{RY}} &= \mathbf{x}_{it}^T \hat{\boldsymbol{\beta}} + \left(\hat{\sigma}_v^2 \mathbf{1}_t^T + \hat{\sigma}_\varepsilon^2 \hat{\gamma}_t \right) \left[\hat{\sigma}_v^2 \mathbf{J}_t + \hat{\sigma}_\varepsilon^2 \hat{\Gamma} + \boldsymbol{\Sigma}_i \right]^{-1} (\bar{\mathbf{y}}_i - \mathbf{X}_i \hat{\boldsymbol{\beta}}) \\ \hat{\boldsymbol{\beta}} &= (\mathbf{X}^T \boldsymbol{\Omega}^{-1} \mathbf{X})^{-1} (\mathbf{X}^T \boldsymbol{\Omega}^{-1} \bar{\mathbf{y}}) \end{aligned}$$

Multivariate Rao-Yu

Variabel respon yang diamati berjumlah lebih dari satu dan saling berkorelasi dengan komponen pengaruh acak area-waktu mengikuti proses autoregresif ordo pertama

Model *Multivariate Rao-Yu* (Jedrzejczak, et al., 2017)

$$\left. \begin{aligned} \theta_{it,d} &= \mathbf{x}_{it,d}^T \boldsymbol{\beta}_d + v_{i,d} + u_{it,d} \\ u_{it} &= \rho u_{i,t-1,k} + \varepsilon_{it,k}, \quad |\rho| < 1, \\ i &= 1, \dots, m, \quad t = 1, \dots, T \end{aligned} \right\} \begin{aligned} y_{it,d} &= \mathbf{x}_{it,d}^T \boldsymbol{\beta}_d + v_{i,d} + u_{it,d} + e_{it,d} \end{aligned}$$

Estimator EBLUP

$$\begin{aligned} \hat{\theta}_{it,d} &= \mathbf{x}_{it,d}^T \hat{\boldsymbol{\beta}}_d + \mathbf{I}_i^T \mathbf{G}_{i,d} \boldsymbol{\Omega}_{i,d}^{-1} (\mathbf{y}_{i,d} - \mathbf{X}_{i,d} \hat{\boldsymbol{\beta}}_d) \\ \hat{\boldsymbol{\beta}} &= (\mathbf{X}^T \boldsymbol{\Omega}^{-1} \mathbf{X})^{-1} \mathbf{X}^T \boldsymbol{\Omega}^{-1} \mathbf{y} \end{aligned}$$

ESTIMASI PARAMETER

Simultaneous Equation Mixed Model (SEMM)

Model persamaan simultan dengan menambahkan komponen *random effects*.

Struktur SEMM adalah sebagai berikut (Sainath, 2014):

$$\begin{bmatrix} y_{11} & y_{12} & \dots & y_{1D} \\ y_{21} & y_{22} & \dots & y_{2D} \\ \vdots & \vdots & \ddots & \vdots \\ y_{m1} & y_{m2} & \dots & y_{mD} \end{bmatrix} \begin{bmatrix} 1 & a_{12} & \dots & a_{1D} \\ a_{21} & 1 & \dots & a_{2D} \\ \vdots & \vdots & \ddots & \vdots \\ a_{D1} & a_{D2} & \dots & 1 \end{bmatrix} =$$

$$\begin{bmatrix} x_{11} & x_{12} & \dots & x_{1q} \\ x_{21} & x_{22} & \dots & x_{2q} \\ \vdots & \vdots & \ddots & \vdots \\ x_{m1} & x_{m2} & \dots & x_{mq} \end{bmatrix} \begin{bmatrix} \beta_{11} & \beta_{12} & \dots & \beta_{1D} \\ \beta_{21} & \beta_{22} & \dots & \beta_{2D} \\ \vdots & \vdots & \ddots & \vdots \\ \beta_{q1} & \beta_{q2} & \dots & \beta_{qD} \end{bmatrix} +$$

$$\begin{bmatrix} z_{11} & z_{12} & \dots & z_{1m} \\ z_{21} & z_{22} & \dots & z_{2m} \\ \vdots & \vdots & \ddots & \vdots \\ z_{m1} & z_{m2} & \dots & z_{mm} \end{bmatrix} \begin{bmatrix} v_{11} & v_{12} & \dots & v_{1D} \\ v_{21} & v_{22} & \dots & v_{2D} \\ \vdots & \vdots & \ddots & \vdots \\ v_{m1} & v_{m2} & \dots & v_{mD} \end{bmatrix} + \begin{bmatrix} e_{11} & e_{12} & \dots & e_{1D} \\ e_{21} & e_{22} & \dots & e_{2D} \\ \vdots & \vdots & \ddots & \vdots \\ e_{m1} & e_{m2} & \dots & e_{mD} \end{bmatrix}$$

$$\longrightarrow \mathbf{YA} = \mathbf{XB} + \mathbf{ZV} + \mathbf{E}, \quad \mathbf{V} \sim N(\mathbf{0}, \mathbf{G}_v), \quad \mathbf{E} \sim N(\mathbf{0}, \mathbf{R}_e)$$

Persamaan dapat ditulis dalam bentuk lain:

$$\mathbf{Y} = \mathbf{YA}^* + \mathbf{XB} + \mathbf{ZV} + \mathbf{E},$$

Untuk persamaan ke- d

$$\mathbf{y}_d = \mathbf{Y}_{(d)} \boldsymbol{\alpha}_{(d)} + \mathbf{X}_d \boldsymbol{\beta}_d + \mathbf{Zv}_d + \mathbf{e}_d, \quad d = 1, \dots, D$$

Dalam bentuk lain

$$\mathbf{y} = \mathbf{X}_A \boldsymbol{\beta}_A + \mathbf{Zv} + \mathbf{e}, \quad \mathbf{v} \sim N(\mathbf{0}, \mathbf{G}(\delta)), \quad \mathbf{e} \sim N(\mathbf{0}, \mathbf{R}(\delta))$$

dimana $\mathbf{y} = \text{col}_{1 \leq d \leq D}(\mathbf{y}_d)$ dengan $\mathbf{y}_d = (y_{1d}, \dots, y_{md})^T$

$$\mathbf{X}_A = \text{diag}_{1 \leq d \leq D}(\mathbf{Y}_{(d)} : \mathbf{X}_d)$$

$$\boldsymbol{\beta}_A = \text{col}_{1 \leq d \leq D}(\boldsymbol{\alpha}_{(d)} : \boldsymbol{\beta}_d)$$

ESTIMASI PARAMETER

Simultaneous Equation Fay Herriot (SEFH)

- Pengembangan dari model MFH (Benavent dan Morales, 2016) dan SEMM (Sainath, 2014).
- Mempertimbangkan adanya hubungan yang saling mempengaruhi antara beberapa variabel respon (Ubaidillah, 2017).

Model SEFH

$$\mathbf{Y} = \mathbf{Y}\mathbf{A}^* + \mathbf{X}\mathbf{B} + \mathbf{Z}\mathbf{V} + \mathbf{E},$$

Untuk persamaan ke- d

$$\mathbf{y}_d = \mathbf{Y}_{(d)}\mathbf{a}_{(d)} + \mathbf{X}_d\mathbf{\beta}_d + \mathbf{Z}\mathbf{v}_d + \mathbf{e}_d, \quad d = 1, \dots, D$$

Dalam bentuk matriks

$$\mathbf{y} = \mathbf{X}_A\mathbf{\beta}_A + \mathbf{Z}\mathbf{v} + \mathbf{e}, \quad \mathbf{v} \sim N(\mathbf{0}, \mathbf{G}(\delta)), \quad \mathbf{e} \sim N(\mathbf{0}, \mathbf{R})$$

$$\mathbf{X}_A = \text{col}_{1 \leq d \leq D}(\mathbf{X}_i)$$

$$\mathbf{X}_i = \text{diag}(y_{i(1)} : x_{i(1)}, \dots, y_{i(D)} : x_{i(D)})$$

Model SEFH untuk unit ke- j

$$y_j = \mathbf{\ddot{x}}_j^T \mathbf{\beta} + \mathbf{z}_j^T \mathbf{v} + e_j, \quad \mathbf{v} \sim N(\mathbf{0}, \mathbf{G}), \quad e_j \sim N(0, \mathbf{R}_j), \quad j = 1, \dots, D_m$$

$$\theta_j = \mathbf{\ddot{x}}_j^T \mathbf{\beta} + \mathbf{z}_j^T \mathbf{v}, \quad j = 1, \dots, D_m$$



**Estimator EBLUP
(3SLS-REML)**

$$\hat{\boldsymbol{\theta}}_{SEFH} = \mathbf{X}_A \hat{\boldsymbol{\beta}}_{A(3SLS-REML)} + \mathbf{Z}\mathbf{G}(\hat{\boldsymbol{\delta}})\mathbf{Z}^T\boldsymbol{\Omega}^{-1}(\hat{\boldsymbol{\delta}})[\mathbf{y} - \mathbf{X}_A \hat{\boldsymbol{\beta}}_{A(3SLS-REML)}]$$

$$\hat{\boldsymbol{\beta}}_{A(3SLS-REML)} = \left(\hat{\mathbf{X}}_A^T \boldsymbol{\Omega}^{-1}(\hat{\boldsymbol{\delta}}) \hat{\mathbf{X}}_A \right)^{-1} \hat{\mathbf{X}}_A^T \boldsymbol{\Omega}^{-1}(\hat{\boldsymbol{\delta}}) \mathbf{y},$$

MODEL SERY



Model SERY



Spesifikasi Model SERY

Model Rao-Yu:

$$y = X\beta + Zw + e,$$

Model SEFH:

$$YA = XB + ZV + E,$$

Model SERY:

$$YA = XB + ZW + E,$$

Estimasi parameter
menggunakan

3SLS-REML

dengan metode iterasi

FSA

Model SERY



Spesifikasi Model SERY

$$\begin{bmatrix} y_1 \\ y_2 \\ \vdots \\ y_D \end{bmatrix} = \begin{bmatrix} Y_{(1)} & 0 & \dots & 0 \\ 0 & Y_{(2)} & \dots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \dots & Y_{(D)} \end{bmatrix} \begin{bmatrix} \alpha_{(1)} \\ \alpha_{(2)} \\ \vdots \\ \alpha_{(D)} \end{bmatrix} + \begin{bmatrix} X_1 & 0 & \dots & 0 \\ 0 & X_2 & \dots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \dots & X_D \end{bmatrix} \begin{bmatrix} \beta_1 \\ \beta_2 \\ \vdots \\ \beta_D \end{bmatrix} + \begin{bmatrix} Z_1 & 0 & \dots & 0 \\ 0 & Z_2 & \dots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \dots & Z_D \end{bmatrix} \begin{bmatrix} w_1 \\ w_2 \\ \vdots \\ w_D \end{bmatrix} + \begin{bmatrix} e_1 \\ e_2 \\ \vdots \\ e_D \end{bmatrix}$$

$$= \begin{bmatrix} Y_{(1)} : X_1 & 0 & \dots & 0 \\ 0 & Y_{(2)} : X_2 & \dots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \dots & Y_{(D)} : X_D \end{bmatrix} \begin{bmatrix} \alpha_{(1)} \\ \beta_1 \\ \alpha_{(2)} \\ \beta_2 \\ \vdots \\ \alpha_{(D)} \\ \beta_D \end{bmatrix}$$

$$+ \begin{bmatrix} Z_1 & 0 & \dots & 0 \\ 0 & Z_2 & \dots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \dots & Z_D \end{bmatrix} \begin{bmatrix} w_1 \\ w_2 \\ \vdots \\ w_D \end{bmatrix} + \begin{bmatrix} e_1 \\ e_2 \\ \vdots \\ e_D \end{bmatrix}$$

Model SERY adalah sebagai berikut::

$$y = X_A \beta_A + Z w + e,$$

dengan

$$y = (y'_1, \dots, y'_D)', \quad Z = I_D \otimes Z_d,$$

$$X_A = (X'_{A_1}, \dots, X'_{A_D})', \quad w = (w'_1, \dots, w'_D)',$$

$$e = (e'_1, \dots, e'_D)'$$

Model SERY merupakan model linier campuran yang mencakup beberapa area kecil dengan beberapa persamaan. Oleh karena itu, perlu didefinisikan model untuk setiap area kecil ke- i pada persamaan ke- d ($d=1, \dots, D$), sebagai berikut

$$y_d = Y_{(d)} \alpha_{(d)} + X_d \beta_d + Z w_d + e_d$$

Model SERY

Untuk mendapatkan estimator pada model SERY, dengan asumsi $\mathbf{v}$, $\mathbf{u}$ dan $\mathbf{e}$ saling bebas satu dengan yang lainnya, maka tentukan matriks varian kovarian dari $\mathbf{y}$:

$$\begin{aligned}
 E(\mathbf{y}) &= E(\mathbf{X}_d \boldsymbol{\beta}_d + \mathbf{Z}\mathbf{w} + \mathbf{e}) \\
 &= E(\mathbf{X}_d \boldsymbol{\beta}_d) + E(\mathbf{Z}\mathbf{w}) + E(\mathbf{e}) \\
 &= \mathbf{X}_d \boldsymbol{\beta}_d + \mathbf{Z}E(\mathbf{w}) + E(\mathbf{e}) \\
 &= \mathbf{X}_d \boldsymbol{\beta}_d, \\
 \text{cov}(\mathbf{y}) &= E\left((\mathbf{y} - \mathbf{X}_d \boldsymbol{\beta}_d)(\mathbf{y} - \mathbf{X}_d \boldsymbol{\beta}_d)'\right) \\
 &= E\left((\mathbf{Z}\mathbf{w} + \mathbf{e})(\mathbf{Z}\mathbf{w} + \mathbf{e})'\right) \\
 &= E(\mathbf{Z}\mathbf{w}\mathbf{w}'\mathbf{Z}' + \mathbf{Z}\mathbf{w}\mathbf{e}' + \mathbf{Z}'\mathbf{e}\mathbf{w}' + \mathbf{e}\mathbf{e}') \\
 &= E(\mathbf{Z}\mathbf{w}\mathbf{w}'\mathbf{Z}') + E(\mathbf{Z}\mathbf{w}\mathbf{e}') + E(\mathbf{Z}'\mathbf{e}\mathbf{w}') + E(\mathbf{e}\mathbf{e}') \\
 &= \mathbf{Z}E(\mathbf{w}\mathbf{w}')\mathbf{Z}' + \mathbf{Z}E(\mathbf{w}\mathbf{e}') + \mathbf{Z}'E(\mathbf{e}\mathbf{w}') + E(\mathbf{e}\mathbf{e}') \\
 &= \mathbf{Z}\mathbf{G}\mathbf{Z}' + \mathbf{0} + \mathbf{0} + \mathbf{R} \\
 &= \mathbf{Z}\mathbf{G}\mathbf{Z}' + \mathbf{R} = \boldsymbol{\Omega}
 \end{aligned}$$

dengan

$$\begin{aligned}
 \boldsymbol{\Omega} &= \text{Cov}(\mathbf{Z}\mathbf{w}) + \text{Cov}(\mathbf{e}) \\
 &= \mathbf{Z}\mathbf{G}\mathbf{Z}' + \mathbf{R} \\
 &= \mathbf{I}_D \otimes (\mathbf{I}_m \otimes [\sigma_v^2 \mathbf{J}_T + \sigma_\varepsilon^2 \boldsymbol{\Pi} + \sigma_e^2 \mathbf{I}_T])
 \end{aligned}$$



Estimasi BLUP

Estimator BLUP model SERY adalah sebagai berikut.

$$\begin{aligned}
 \tilde{\boldsymbol{\theta}} &= \mathbf{X}_d \tilde{\boldsymbol{\beta}}_d + \mathbf{Z}\tilde{\mathbf{w}} \\
 &= \mathbf{X}_d \tilde{\boldsymbol{\beta}}_d + \mathbf{Z}\mathbf{G}\mathbf{Z}'\boldsymbol{\Omega}^{-1}[\mathbf{y} - \mathbf{X}_d \tilde{\boldsymbol{\beta}}_d],
 \end{aligned}$$

Karena model SERY merupakan model linier campuran yang mencakup beberapa model area, maka untuk estimator BLUP untuk v_i dan u_{it} diturunkan dari model area kecil ke- i persamaan ke- d

$$\begin{aligned}
 \tilde{\mathbf{v}}_{di} + \tilde{\mathbf{u}}_{diT} &= \mathbf{k}'\tilde{\mathbf{w}}_{di} \\
 &= (1, \mathbf{0}_{T-1}', 1) \begin{pmatrix} \sigma_v^2 \mathbf{I}_T' \\ \sigma_\varepsilon^2 \boldsymbol{\Pi} \end{pmatrix} (\sigma_v^2 \mathbf{J}_T + \sigma_\varepsilon^2 \boldsymbol{\Pi} + \sigma_e^2 \mathbf{I}_T)^{-1} (\mathbf{y}_{di} - \mathbf{X}_{Ad} \tilde{\boldsymbol{\beta}}_{Ad}) \\
 &= (\sigma_v^2 \mathbf{I}_T' + \sigma_\varepsilon^2 \boldsymbol{\gamma}_T') (\sigma_v^2 \mathbf{J}_T + \sigma_\varepsilon^2 \boldsymbol{\Pi} + \sigma_e^2 \mathbf{I}_T)^{-1} (\mathbf{y}_{di} - \mathbf{X}_{Ad} \tilde{\boldsymbol{\beta}}_{Ad})
 \end{aligned}$$

$$\begin{aligned}
 \tilde{\mathbf{v}}_{id} &= \mathbf{G}_{1_{id}} \mathbf{Z}_{1_{id}}' \boldsymbol{\Omega}_{id}^{-1} (\mathbf{y}_{id} - \mathbf{X}_{Ad} \tilde{\boldsymbol{\beta}}_{Ad}) \\
 &= \sigma_v^2 \mathbf{1}_T' (\sigma_v^2 \mathbf{J}_T + \sigma_\varepsilon^2 \boldsymbol{\Pi} + \sigma_e^2 \mathbf{I}_T)^{-1} (\mathbf{y}_{id} - \mathbf{X}_{Ad} \tilde{\boldsymbol{\beta}}_{Ad}) \\
 \tilde{\mathbf{u}}_{id} &= \mathbf{G}_{2_{id}} \mathbf{Z}_{2_{id}}' \boldsymbol{\Omega}_{id}^{-1} (\mathbf{y}_{id} - \mathbf{X}_{Ad} \tilde{\boldsymbol{\beta}}_{Ad}) \\
 &= \sigma_\varepsilon^2 \boldsymbol{\Pi} (\sigma_v^2 \mathbf{J}_T + \sigma_\varepsilon^2 \boldsymbol{\Pi} + \sigma_e^2 \mathbf{I}_T)^{-1} (\mathbf{y}_{id} - \mathbf{X}_{Ad} \tilde{\boldsymbol{\beta}}_{Ad})
 \end{aligned}$$

Model SERY

Estimasi BLUP

Estimator BLUP θ_{iTd} untuk setiap area ke- i , waktu ke- t , dan persamaan ke- d

$$\begin{aligned}\tilde{\theta}_{iTd} &= \mathbf{X}_{A_{id}} \tilde{\beta}_{A_d} + \tilde{v}_{id} + \tilde{u}_{iTd} \\ &= \mathbf{X}_{A_{id}} \tilde{\beta}_{A_d} + \left[(\sigma_v^2 \mathbf{1}'_T) (\sigma_v^2 \mathbf{J}_T + \sigma_\varepsilon^2 \mathbf{\Pi} + \sigma_e^2 \mathbf{I}_T)^{-1} \right] (\mathbf{y}_{id} - \mathbf{X}_{A_{id}} \tilde{\beta}_{A_d}) \\ &\quad + \left[(\sigma_\varepsilon^2 \gamma_T) (\sigma_v^2 \mathbf{J}_T + \sigma_\varepsilon^2 \mathbf{\Pi} + \sigma_e^2 \mathbf{I}_T)^{-1} \right] (\mathbf{y}_{id} - \mathbf{X}_{A_{id}} \tilde{\beta}_{A_d}) \\ &= \mathbf{X}_{A_{id}} \tilde{\beta}_{A_d} + \left[(\sigma_v^2 \mathbf{1}'_T + \sigma_\varepsilon^2 \gamma_T) (\sigma_v^2 \mathbf{J}_T + \sigma_\varepsilon^2 \mathbf{\Pi} + \sigma_e^2 \mathbf{I}_T)^{-1} \right] (\mathbf{y}_{id} - \mathbf{X}_{A_{id}} \tilde{\beta}_{A_d})\end{aligned}$$

dimana

$$\tilde{v}_{id} = \sigma_v^2 \mathbf{1}'_T (\sigma_v^2 \mathbf{J}_T + \sigma_\varepsilon^2 \mathbf{\Pi} + \sigma_e^2 \mathbf{I}_T)^{-1} (\mathbf{y}_{id} - \mathbf{X}_{A_{id}} \tilde{\beta}_{A_d})$$

$$\tilde{u}_{iTd} = \sigma_\varepsilon^2 \gamma_T (\sigma_v^2 \mathbf{J}_T + \sigma_\varepsilon^2 \mathbf{\Pi} + \sigma_e^2 \mathbf{I}_T)^{-1} (\mathbf{y}_{id} - \mathbf{X}_{A_{id}} \tilde{\beta}_{A_d})$$

Estimasi EBLUP

Langkah 1.

Estimasi β_A menggunakan 2SLS

Ubah persamaan struktural ke dalam persamaan *reduced form*

$$\begin{aligned}\mathbf{Y} &= \mathbf{XBA}^{-1} + \mathbf{ZWA}^{-1} + \mathbf{EA}^{-1}, \\ &= \mathbf{XA} + \mathbf{ZE} + \mathbf{\Theta}\end{aligned}$$

Estimasi Λ menggunakan OLS dan masukkan $\hat{\mathbf{Y}} = \mathbf{X}\hat{\Lambda}$.

Substitusikan $\hat{\mathbf{Y}}$ kedalam $\mathbf{X}$ untuk mendapatkan $\hat{\mathbf{X}}_A$.

Kemudian estimasi β_A menggunakan 2SLS

$$\hat{\beta}_{A_{2SLS}} = (\hat{\mathbf{X}}'_A \hat{\mathbf{X}}_A)^{-1} \hat{\mathbf{X}}'_A \mathbf{y}.$$

Langkah 2.

Estimasi komponen varian menggunakan REML

Fungsi likelihood $\mathbf{y}$ adalah:

$$L_R(\delta) = (2\pi)^{-\frac{DmT - q_A}{2}} |\mathbf{\Omega}|^{-\frac{1}{2}} \left[\exp \left(-\frac{1}{2} (\mathbf{y} - \mathbf{X}_A \beta_A)' \mathbf{\Omega}^{-1} (\mathbf{y} - \mathbf{X}_A \beta_A) \right) \right]$$

Model SERY

Estimasi EBLUP

Transformasi data dinotasikan dengan $\mathbf{C}'\mathbf{y}$, $\mathbf{C}'\mathbf{X}_A = \mathbf{0}_{(DmT-q_A) \times q_A}$ sehingga

$$\mathbf{y} = \mathbf{X}_A \boldsymbol{\beta}_A + \mathbf{Z}\mathbf{w} + \mathbf{e}, \longrightarrow \mathbf{C}'\mathbf{y} = \mathbf{C}'\mathbf{X}_A \boldsymbol{\beta}_A + \mathbf{C}'\mathbf{Z}\mathbf{w} + \mathbf{C}'\mathbf{e},$$

Fungsi likelihood:

$$L_R(\delta) = (2\pi)^{-\frac{DmT-q_A}{2}} |\mathbf{C}'\boldsymbol{\Omega}\mathbf{C}|^{-\frac{1}{2}} \left[\exp\left(-\frac{1}{2}(\mathbf{C}'\mathbf{y})'(\mathbf{C}'\boldsymbol{\Omega}\mathbf{C})^{-1}(\mathbf{C}'\mathbf{y})\right) \right]$$

Fungsi restricted log-likelihood:

$$\ell_R(\delta) = -\frac{DmT-q_A}{2} \log(2\pi) - \frac{1}{2} \log|\mathbf{C}'\boldsymbol{\Omega}\mathbf{C}| - \frac{1}{2}(\mathbf{C}'\mathbf{y})'(\mathbf{C}'\boldsymbol{\Omega}\mathbf{C})^{-1}(\mathbf{C}'\mathbf{y})$$

dimana

$$\log|\mathbf{C}'\boldsymbol{\Omega}\mathbf{C}| = \log|\boldsymbol{\Omega}| + \log|\mathbf{X}'_A \boldsymbol{\Omega}^{-1} \mathbf{X}_A|$$

$$\mathbf{P} = \mathbf{C}(\mathbf{C}'\boldsymbol{\Omega}\mathbf{C})^{-1} \mathbf{C}' = \boldsymbol{\Omega}^{-1} - \boldsymbol{\Omega}^{-1} \mathbf{X}_A (\mathbf{X}'_A \boldsymbol{\Omega}^{-1} \mathbf{X}_A)^{-1} (\mathbf{X}'_A \boldsymbol{\Omega}^{-1}).$$

sehingga

$$\ell_R = -\frac{DmT-q_A}{2} \log(2\pi) - \frac{1}{2} \log|\mathbf{C}'\boldsymbol{\Omega}\mathbf{C}| - \frac{1}{2} \mathbf{y}'\mathbf{P}\mathbf{y}.$$

Estimator komponen varian diperoleh dengan memaksimumkan fungsi likelihood. Turunan parsial pertama dari $\ell_R(\delta)$ terhadap $\delta_j, j=1, \dots, r$ menghasilkan nilai score vector.

$$\begin{aligned} s_j(\delta) &= \frac{\partial}{\partial \delta_j} \left[-\frac{1}{2} \log|\mathbf{C}'\boldsymbol{\Omega}\mathbf{C}| - \frac{1}{2} \mathbf{y}'\mathbf{P}\mathbf{y} \right] \\ &= -\frac{1}{2} \text{tr} \left[(\mathbf{C}'\boldsymbol{\Omega}\mathbf{C})^{-1} \left(\mathbf{C}' \frac{\partial \boldsymbol{\Omega}}{\partial \delta_j} \mathbf{C} \right) \right] - \frac{1}{2} \mathbf{y}' \frac{\partial [\mathbf{C}(\mathbf{C}'\boldsymbol{\Omega}\mathbf{C})^{-1} \mathbf{C}']}{\partial \delta_j} \mathbf{y} \\ &= -\frac{1}{2} \text{tr} [(\mathbf{C}'\boldsymbol{\Omega}\mathbf{C})^{-1} (\mathbf{C}'\boldsymbol{\Omega}_j \mathbf{C})] + \frac{1}{2} \mathbf{y}' \mathbf{C} [(\mathbf{C}'\boldsymbol{\Omega}\mathbf{C})^{-1} (\mathbf{C}'\boldsymbol{\Omega}_j \mathbf{C}) (\mathbf{C}'\boldsymbol{\Omega}\mathbf{C})^{-1}] \mathbf{C}' \mathbf{y} \\ &= -\frac{1}{2} \text{tr} [\mathbf{C}(\mathbf{C}'\boldsymbol{\Omega}\mathbf{C})^{-1} \mathbf{C}'\boldsymbol{\Omega}_j] + \frac{1}{2} \mathbf{y}' [\mathbf{C}(\mathbf{C}'\boldsymbol{\Omega}\mathbf{C})^{-1} \mathbf{C}'\boldsymbol{\Omega}_j \mathbf{C}(\mathbf{C}'\boldsymbol{\Omega}\mathbf{C})^{-1}] \mathbf{y} \\ &= -\frac{1}{2} \text{tr} (\mathbf{P}\boldsymbol{\Omega}_{(j)}) + \frac{1}{2} \mathbf{y}' \mathbf{P}\boldsymbol{\Omega}_{(j)} \mathbf{y}, \end{aligned}$$

sehingga diperoleh

$$s_1(\delta) = \frac{\partial \ell_R(\delta)}{\partial \sigma_v^2} = -\frac{1}{2} \text{tr} (\mathbf{P}(\mathbf{I}_D \otimes (\mathbf{I}_m \otimes \mathbf{J}_T))) + \frac{1}{2} \mathbf{y}' \mathbf{P}(\mathbf{I}_D \otimes (\mathbf{I}_m \otimes \mathbf{J}_T)) \mathbf{y},$$

$$s_2(\delta) = \frac{\partial \ell_R(\delta)}{\partial \sigma_\varepsilon^2} = -\frac{1}{2} \text{tr} (\mathbf{P}(\mathbf{I}_D \otimes (\mathbf{I}_m \otimes \boldsymbol{\Pi}))) + \frac{1}{2} \mathbf{y}' \mathbf{P}(\mathbf{I}_D \otimes (\mathbf{I}_m \otimes \boldsymbol{\Pi})) \mathbf{y},$$

$$s_3(\delta) = \frac{\partial \ell_R(\delta)}{\partial \rho} = -\frac{1}{2} \text{tr} \left(\sigma_\varepsilon^2 \mathbf{P} \left(\mathbf{I}_D \otimes \left(\mathbf{I}_m \otimes \frac{\partial \boldsymbol{\Pi}}{\partial \rho} \right) \right) \right) + \frac{1}{2} \sigma_\varepsilon^2 \mathbf{y}' \mathbf{P} \left(\mathbf{I}_D \otimes \left(\mathbf{I}_m \otimes \frac{\partial \boldsymbol{\Pi}}{\partial \rho} \right) \right) \mathbf{y}.$$

Model SERY

Estimasi EBLUP

Iterasi *Fisher Scoring Algorithm*, penyelesaian sampai dengan iterasi ke- $(a+1)$

$$\hat{\delta}^{(a+1)} = \hat{\delta}^{(a)} + \mathbf{F}_{j,l}^{-1} \left(\hat{\delta}^{(a)} \right) \mathbf{s} \left(\hat{\delta}^{(a)} \right), \quad j, l = 1, \dots, r$$

dimana

$$\mathbf{F}_{j,l}(\delta) = E \left[-\frac{\partial^2 \ell_R(\delta)}{\partial \delta_j \partial \delta_l} \right] = \frac{1}{2} \text{tr} \left(\mathbf{P} \mathbf{\Omega}_{(l)} \mathbf{P} \mathbf{\Omega}_{(j)} \right), \quad k, l = 1, \dots, r$$

dengan

$$\mathbf{F}_{11} = \frac{1}{2} \text{tr} \left(\mathbf{P} \frac{\partial \mathbf{\Omega}}{\partial \sigma_v^2} \mathbf{P} \frac{\partial \mathbf{\Omega}}{\partial \sigma_v^2} \right) = \frac{1}{2} \text{tr} \left(\mathbf{P} (\mathbf{I}_D \otimes (\mathbf{I}_m \otimes \mathbf{J}_T)) \mathbf{P} (\mathbf{I}_D \otimes (\mathbf{I}_m \otimes \mathbf{J}_T)) \right),$$

$$\mathbf{F}_{12} = \mathbf{F}_{21} = \frac{1}{2} \text{tr} \left(\mathbf{P} \frac{\partial \mathbf{\Omega}}{\partial \sigma_v^2} \mathbf{P} \frac{\partial \mathbf{\Omega}}{\partial \sigma_\varepsilon^2} \right) = \frac{1}{2} \text{tr} \left(\mathbf{P} (\mathbf{I}_D \otimes (\mathbf{I}_m \otimes \mathbf{J}_T)) \mathbf{P} (\mathbf{I}_D \otimes (\mathbf{I}_m \otimes \mathbf{\Pi})) \right),$$

$$\mathbf{F}_{13} = \mathbf{F}_{31} = \frac{1}{2} \text{tr} \left(\mathbf{P} \frac{\partial \mathbf{\Omega}}{\partial \sigma_v^2} \mathbf{P} \frac{\partial \mathbf{\Omega}}{\partial \rho} \right) = \frac{1}{2} \text{tr} \left(\mathbf{P} (\mathbf{I}_D \otimes (\mathbf{I}_m \otimes \mathbf{J}_T)) \sigma_\varepsilon^2 \mathbf{P} \left(\mathbf{I}_D \otimes \left(\mathbf{I}_m \otimes \frac{\partial \mathbf{\Pi}}{\partial \rho} \right) \right) \right),$$

$$\mathbf{F}_{22} = \frac{1}{2} \text{tr} \left(\mathbf{P} \frac{\partial \mathbf{\Omega}}{\partial \sigma_\varepsilon^2} \mathbf{P} \frac{\partial \mathbf{\Omega}}{\partial \sigma_\varepsilon^2} \right) = \frac{1}{2} \text{tr} \left(\mathbf{P} (\mathbf{I}_D \otimes (\mathbf{I}_m \otimes \mathbf{\Pi})) \mathbf{P} (\mathbf{I}_D \otimes (\mathbf{I}_m \otimes \mathbf{\Pi})) \right),$$

$$\mathbf{F}_{23} = \mathbf{F}_{32} = \frac{1}{2} \text{tr} \left(\mathbf{P} \frac{\partial \mathbf{\Omega}}{\partial \sigma_\varepsilon^2} \mathbf{P} \frac{\partial \mathbf{\Omega}}{\partial \rho} \right) = \frac{1}{2} \text{tr} \left(\mathbf{P} (\mathbf{I}_D \otimes (\mathbf{I}_m \otimes \mathbf{\Pi})) \sigma_\varepsilon^2 \mathbf{P} \left(\mathbf{I}_D \otimes \left(\mathbf{I}_m \otimes \frac{\partial \mathbf{\Pi}}{\partial \rho} \right) \right) \right)$$

$$\mathbf{F}_{33} = \frac{1}{2} \text{tr} \left(\mathbf{P} \frac{\partial \mathbf{\Omega}}{\partial \rho} \mathbf{P} \frac{\partial \mathbf{\Omega}}{\partial \rho} \right) = \frac{1}{2} \text{tr} \left(\sigma_\varepsilon^2 \mathbf{P} \left(\mathbf{I}_D \otimes \left(\mathbf{I}_m \otimes \frac{\partial \mathbf{\Pi}}{\partial \rho} \right) \right) \sigma_\varepsilon^2 \mathbf{P} \left(\mathbf{I}_D \otimes \left(\mathbf{I}_m \otimes \frac{\partial \mathbf{\Pi}}{\partial \rho} \right) \right) \right).$$

Langkah 3.

Estimasi β_A menggunakan metode GLS

$$\hat{\beta}_{A_{3SLS-REML}} = \left(\hat{\mathbf{X}}'_A \mathbf{\Omega}^{-1} \hat{\mathbf{X}}_A \right)^{-1} \hat{\mathbf{X}}'_A \mathbf{\Omega}^{-1} \mathbf{y},$$

Langkah 4.

Substitusikan hasil estimasi parameter ke dalam estimator BLUP
Sehingga diperoleh estimator EBLUP

$$\hat{\theta}_{SERY} = \mathbf{X}_A \hat{\beta}_{A_{3SLS-REML}} + \mathbf{ZGZ}' \mathbf{\Omega}^{-1} \left[\mathbf{y} - \mathbf{X}_A \hat{\beta}_{A_{3SLS-REML}} \right].$$

Model SERY

Estimasi MSE

Estimasi MSE dari model SERY diperoleh dengan mengambil elemen diagonal matriks kovarians EBLUP

$$\begin{aligned}\text{cov}(\hat{\boldsymbol{\theta}}) &= E\left[(\hat{\boldsymbol{\theta}} - \boldsymbol{\theta})(\hat{\boldsymbol{\theta}} - \boldsymbol{\theta})'\right] \\ &= E\left[(\tilde{\boldsymbol{\theta}} - \boldsymbol{\theta})(\tilde{\boldsymbol{\theta}} - \boldsymbol{\theta})'\right] + E\left[(\hat{\boldsymbol{\theta}} - \tilde{\boldsymbol{\theta}})(\hat{\boldsymbol{\theta}} - \tilde{\boldsymbol{\theta}})'\right] + 2E\left[(\tilde{\boldsymbol{\theta}} - \boldsymbol{\theta})(\hat{\boldsymbol{\theta}} - \tilde{\boldsymbol{\theta}})'\right] \\ &= \boldsymbol{\Sigma}_1 + \boldsymbol{\Sigma}_2 + 2\boldsymbol{\Sigma}_3\end{aligned}$$

dimana $\boldsymbol{\Sigma}_1$ adalah matriks kovarian BLUP. Misalkan $\boldsymbol{\Gamma} = \mathbf{Z}\mathbf{G}\mathbf{Z}'\boldsymbol{\Omega}^{-1}$,

$$\boldsymbol{\Sigma}_1 = \text{cov}(\tilde{\boldsymbol{\theta}}) = \boldsymbol{\Gamma}\mathbf{R} + (\mathbf{I} - \boldsymbol{\Gamma})\mathbf{X}_A(\mathbf{X}_A'\boldsymbol{\Omega}^{-1}\mathbf{X}_A)^{-1}\mathbf{X}_A'(\mathbf{I} - \boldsymbol{\Gamma})'$$

$$\begin{aligned}\boldsymbol{\Sigma}_2 &= E\left[(\hat{\boldsymbol{\theta}} - \tilde{\boldsymbol{\theta}})(\hat{\boldsymbol{\theta}} - \tilde{\boldsymbol{\theta}})'\right] \\ &= E\left\{\frac{\partial \boldsymbol{\Gamma}}{\partial \boldsymbol{\delta}}(\mathbf{y} - \mathbf{X}_A\boldsymbol{\beta}_A) + \frac{\partial}{\partial \boldsymbol{\delta}}\left[(\mathbf{I} - \boldsymbol{\Gamma})\mathbf{X}_A(\tilde{\boldsymbol{\beta}}_A - \boldsymbol{\beta}_A)(\hat{\boldsymbol{\theta}} - \boldsymbol{\theta})\right]\right\} \\ &\approx \sum_{j=1}^r \sum_{l=1}^r \text{cov}(\hat{\delta}_j, \hat{\delta}_l) \boldsymbol{\Gamma}_{(j)} \boldsymbol{\Omega} \boldsymbol{\Gamma}'_{(l)}.\end{aligned}$$

$$\begin{aligned}\boldsymbol{\Sigma}_3 &= E\left[(\tilde{\boldsymbol{\theta}} - \boldsymbol{\theta})(\hat{\boldsymbol{\theta}} - \tilde{\boldsymbol{\theta}})'\right] \\ &= E\left[\mathbf{X}_A\boldsymbol{\beta}_A + \boldsymbol{\Gamma}(\mathbf{y} - \mathbf{X}_A\boldsymbol{\beta}_A) + (\mathbf{I} - \boldsymbol{\Gamma})\mathbf{X}_A(\tilde{\boldsymbol{\beta}}_A - \boldsymbol{\beta}_A) - \mathbf{X}_A\boldsymbol{\beta}_A - \mathbf{Z}_1\mathbf{v} - \mathbf{Z}_2\mathbf{u}\right](\hat{\boldsymbol{\theta}} - \tilde{\boldsymbol{\theta}})' \\ &= \mathbf{0}\end{aligned}$$

$$\begin{aligned}\text{cov}(\hat{\boldsymbol{\theta}}) &\approx \boldsymbol{\Gamma}\mathbf{R} + (\mathbf{I} - \boldsymbol{\Gamma})\mathbf{X}_A(\mathbf{X}_A'\boldsymbol{\Omega}^{-1}\mathbf{X}_A)^{-1}\mathbf{X}_A'(\mathbf{I} - \boldsymbol{\Gamma})' + \sum_{j=1}^r \sum_{l=1}^r \text{cov}(\hat{\delta}_j, \hat{\delta}_l) \boldsymbol{\Gamma}_{(j)} \boldsymbol{\Omega} \boldsymbol{\Gamma}'_{(l)} \\ &\approx \boldsymbol{\Theta}_1(\boldsymbol{\delta}) + \boldsymbol{\Theta}_2(\boldsymbol{\delta}) + \boldsymbol{\Theta}_3(\boldsymbol{\delta})\end{aligned}$$

dimana

$$\boldsymbol{\Theta}_1(\boldsymbol{\delta}) = \boldsymbol{\Gamma}\mathbf{R},$$

$$\boldsymbol{\Theta}_2(\boldsymbol{\delta}) = (\mathbf{I} - \boldsymbol{\Gamma})\mathbf{X}_A(\mathbf{X}_A'\boldsymbol{\Omega}^{-1}\mathbf{X}_A)^{-1}\mathbf{X}_A'(\mathbf{I} - \boldsymbol{\Gamma})',$$

$$\boldsymbol{\Theta}_3(\boldsymbol{\delta}) \approx \sum_{j=1}^r \sum_{l=1}^r \text{cov}(\hat{\delta}_j, \hat{\delta}_l) \boldsymbol{\Gamma}_{(j)} \boldsymbol{\Omega} \boldsymbol{\Gamma}'_{(l)}, \quad j, l = 1, \dots, r$$

MSE EBLUP mengambil elemen diagonal dari matriks kovarian $\text{cov}(\hat{\boldsymbol{\theta}})$

$$MSE(\hat{\theta}) = \text{col}_{1 \leq h \leq DmT} \left[\text{cov}(\hat{\boldsymbol{\theta}}_{\text{SERY}}) \right]_h, \quad h = 1, \dots, DmT,$$

$$MSE(\hat{\boldsymbol{\theta}}) \approx \mathbf{g}_1(\boldsymbol{\delta}) + \mathbf{g}_2(\boldsymbol{\delta}) + \mathbf{g}_3(\boldsymbol{\delta}),$$

$$\text{dimana } \mathbf{g}_b(\boldsymbol{\delta}) = \text{col}_{1 \leq h \leq DmT} \left[\boldsymbol{\Theta}_b(\boldsymbol{\delta}) \right]_h, \quad h = 1, \dots, DmT, \quad b = 1, 2, 3$$

Estimator $MSE(\hat{\boldsymbol{\theta}})$ adalah

$$mse(\hat{\boldsymbol{\theta}}) \approx \mathbf{g}_1(\hat{\boldsymbol{\delta}}) + \mathbf{g}_2(\hat{\boldsymbol{\delta}}) + 2\mathbf{g}_3(\hat{\boldsymbol{\delta}}).$$

SIMULASI MODEL



Simulasi

Persamaan model SERY yang dirancang untuk simulasi ini berdasarkan penelitian Noviyanti, dkk (2024).

$$y_{1it} = \alpha_{21}y_{2it} + \beta_{01} + \beta_{11}x_{1it} + \beta_{21}x_{2it} + \beta_{31}x_{3it} + v_{i1} + u_{it1} + e_{it1}$$
$$y_{2it} = \alpha_{12}y_{1it} + \beta_{02} + \beta_{12}x_{1it} + \beta_{42}x_{4it} + \beta_{52}x_{5it} + v_{i2} + u_{it2} + e_{it2}$$

Evaluasi:



Bias model



Efisiensi model

Skenario:

1

Area

$m = 10, 30, 50$

2

Waktu

$t = 3, 5, 8$

Tujuan 1. Evaluasi bias model

Tabel 5.1 Nilai Bias dan RMSE Hasil Simulasi dengan skenario $m=10, 30, 50$ pada $t=5$

Output	Variabel	$t=5$		
		$m=10$	$m=30$	$m=50$
Bias	Y_1	-0,19854	-0,14289	-0,01654
	Y_2	-0,02710	0,02075	-0,00855
RMSE	Y_1	1,41862	1,02574	0,79754
	Y_2	1,26550	1,02574	0,63553

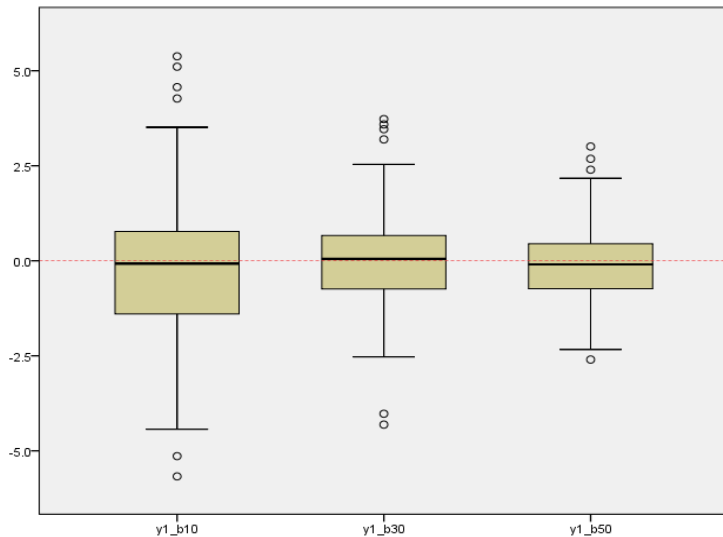
Tabel 5.2 Nilai Bias dan RMSE Hasil Simulasi dengan skenario $t=3, 5, 8$ pada $m=30$

Output	Variabel	$m=30$		
		$t=3$	$t=5$	$t=8$
Bias	Y_1	-0,15100	-0,14289	0,03417
	Y_2	0,18196	0,02075	-0,01086
RMSE	Y_1	1,31084	0,92299	0,70043
	Y_2	1,06711	0,81299	0,56715

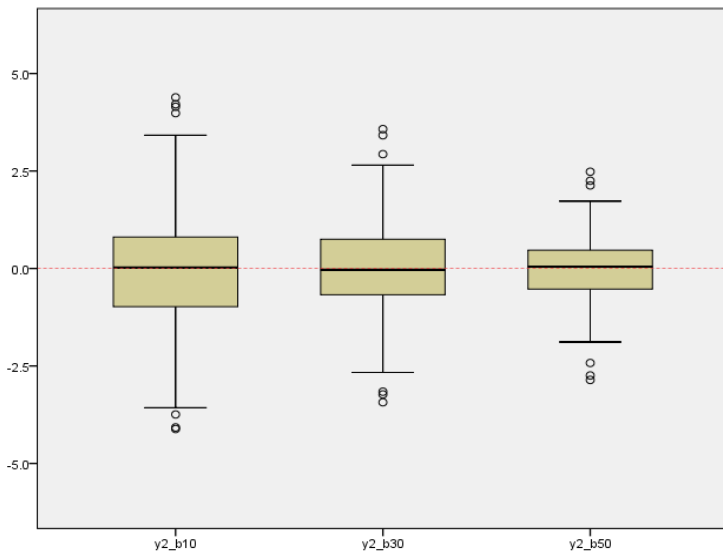
Simulasi

Skenario area (Data simulasi $m=10, 30, 50$ dengan $t=5$)

Gambar 5.1
Boxplot bias hasil estimasi
variabel Y_1

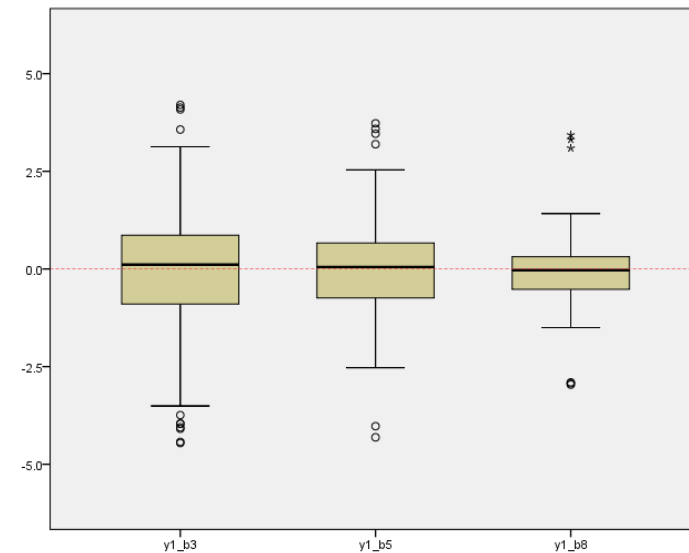


Gambar 5.2
Boxplot bias hasil estimasi
variabel Y_2

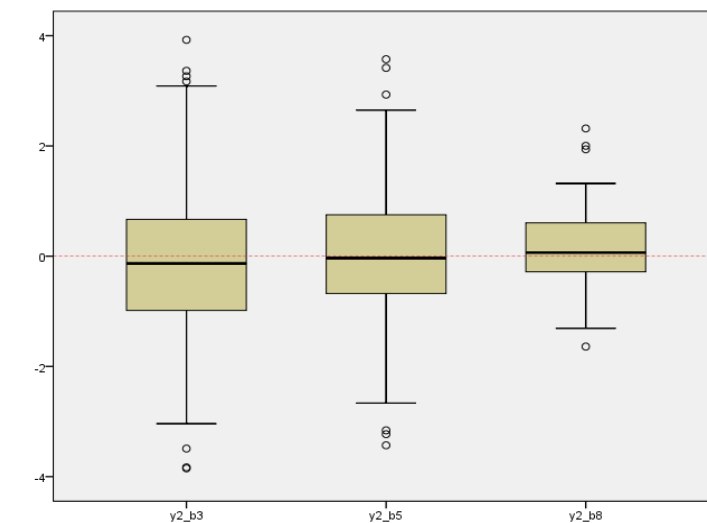


Skenario waktu (Data simulasi $t=3, 5, 8$ dengan $m=30$)

Gambar 5.3
Boxplot bias hasil estimasi
variabel Y_1



Gambar 5.4
Boxplot bias hasil estimasi
variabel Y_2

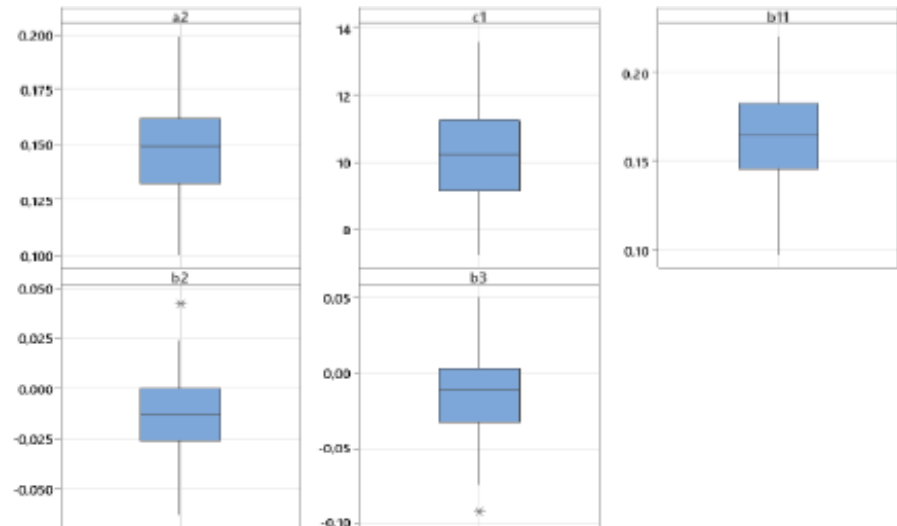


Simulasi

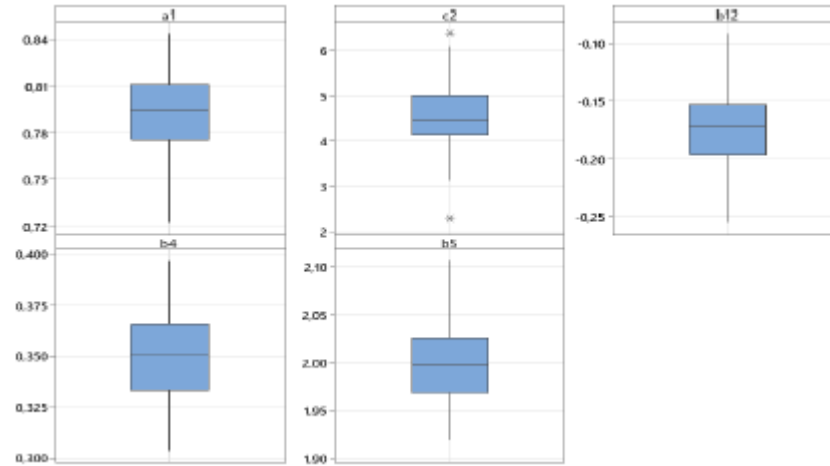
Tujuan 2. Evaluasi estimator parameter model

Replikasi 100 Kali

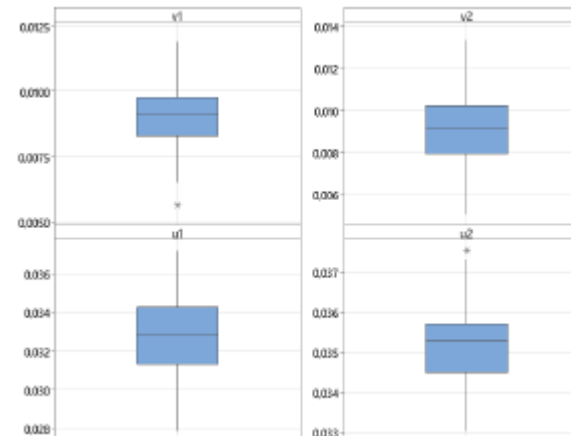
Boxplot Koefisien Parameter Y1 :



Boxplot Koefisien Parameter Y2 :



Boxplot Komponen Varian:



Simulasi

Tujuan 2. Evaluasi estimator parameter model

Tabel 5.3 Selang Kepercayaan Koefisien Parameter Model SERY
Data Simulasi ($m=30$ dan $t=5$)

Parameter	$\bar{\hat{\beta}}_A$	True Parameter	Selang Kepercayaan	
			5%	95%
α_{21}	0,10188	0,1	0,09864	0,10513
β_{01}	10,23599	10	9,99549	10,47648
β_{11}	0,10226	0,1	0,09757	0,10694
β_{21}	0,11992	-0,01	0,11608	0,12376
β_{31}	0,04851	-0,01	0,01407	0,08294
α_{12}	0,76238	0,7	0,71969	0,80507
β_{02}	4,15529	4	4,01824	4,29234
β_{12}	0,11059	-0,1	0,10545	0,11572
β_{42}	0,35019	-0,3	0,34670	0,35368
β_{52}	2,28993	2,3	2,28220	2,29767

Tabel 5.4 Selang Kepercayaan Komponen Varian
Data Simulasi ($m=30$ dan $t=5$)

Parameter	$\overline{\hat{\sigma}^2}$	True Parameter	Selang Kepercayaan	
			5%	95%
$\sigma_{v_{11}}^2$	0,00920	0,01	0,00553	0,01287
$\sigma_{v_{22}}^2$	0,00914	0,01	0,00668	0,01160
$\sigma_{\varepsilon_{11}}^2$	0,03081	0,03	0,03054	0,03108
$\sigma_{\varepsilon_{22}}^2$	0,03015	0,03	0,02890	0,03140

Model dengan bias rendah dan varians rendah akan menghasilkan estimasi yang konsisten dan akurat.

PENERAPAN MODEL



Penerapan Model

Penerapan Model SERY

Variabel Penelitian

Tabel 3.1 Variabel Penelitian dan Definisi Operasional

Variabel	Nama Variabel	Definisi Operasional
Y_1	Rata-rata pengeluaran per kapita makanan.	Rata-rata biaya yang dikeluarkan untuk konsumsi makanan semua anggota rumah tangga selama sebulan dibagi dengan banyaknya ART.
Y_2	Rata-rata pengeluaran per kapita non-makanan.	Rata-rata biaya yang dikeluarkan untuk konsumsi bukan makanan semua anggota rumah tangga selama sebulan dibagi dengan banyaknya ART.
X_1	Jumlah penduduk.	Semua orang yang berdomisili di wilayah geografis Indonesia selama 6 bulan atau lebih dan atau mereka yang berdomisili kurang dari 6 bulan tetapi bertujuan untuk menetap.
X_2	Luas panen produksi tanaman pangan.	Luas panen produksi tanaman padi sawah dan padi ladang di setiap kecamatan.
X_3	Jumlah industri.	Jumlah industri mikro dan kecil di setiap kecamatan.
X_4	Rasio fasilitas kesehatan terhadap 1000 penduduk.	Jumlah fasilitas kesehatan di setiap kecamatan per 1000 penduduk.
X_5	Rasio fasilitas pendidikan per 1000 penduduk.	Jumlah sekolah SD, SMP, SMA dan SMK baik negeri maupun swasta di setiap kecamatan per 1000 penduduk.

Sumber Data

1. Data karakteristik individu yang digunakan untuk estimasi Y_1 dan Y_2 berdasarkan data hasil Susenas tahun 2018 – 2022 yang diselenggarakan di Kabupaten Langkat.
2. Data karakteristik wilayah kecamatan di Kabupaten Langkat yang meliputi variabel dan seperti yang dijelaskan pada tabel 3.1 bersumber dari Daerah dalam Angka.

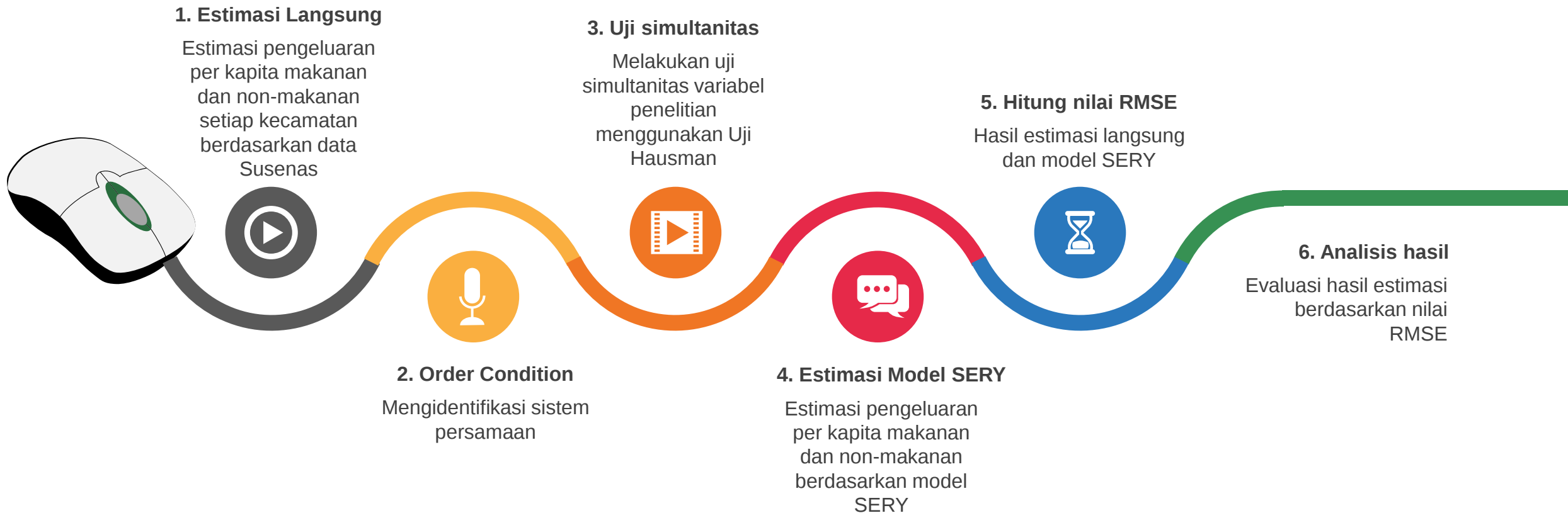
Struktur Data

Area (i)	Waktu (t)	Variabel respon/endogen (y_{it})				Variabel penyerta/eksogen (x_{it})			
		y_1	y_2	...	y_p	x_1	x_2	...	x_q
1	1	y_{111}	y_{211}		y_{p11}	x_{111}	x_{211}		x_{q11}
	2	y_{112}	y_{212}		y_{p12}	x_{112}	x_{212}		x_{q12}
	$\vdots$								
	T	y_{11T}	y_{21T}		y_{p1T}	x_{11T}	x_{21T}		x_{q1T}
2	1	y_{121}	y_{221}		y_{p21}	x_{121}	x_{221}		x_{q21}
	2	y_{122}	y_{222}		y_{p22}	x_{122}	x_{222}		x_{q22}
	$\vdots$								
	T	y_{12T}	y_{22T}		y_{p2T}	x_{12T}	x_{22T}		x_{q2T}
$\vdots$									
m	1	y_{1m1}	y_{2m1}		$y_{p m1}$	x_{1m1}	x_{2m1}		$x_{q m1}$
	2	y_{1m2}	y_{2m2}		$y_{p m2}$	x_{1m2}	x_{2m2}		$x_{q m2}$
	$\vdots$								
	T	y_{1mT}	y_{2mT}		$y_{p mT}$	x_{1mT}	x_{2mT}		$x_{q mT}$

Penerapan Model

Penerapan Model SERY

Tahapan dalam penerapan model:



Penerapan Model

Estimasi Langsung

Rumus pengeluaran per kapita:

$$\hat{y}_{it} = \frac{\sum_{n \in \omega_{it}} \omega_{in} y_{itn}}{N_{it}}$$

dimana

$\hat{y}_{it}$ = pengeluaran per kapita kecamatan ke- i tahun ke- t .

y_{itn} = pengeluaran per kapita rumah tangga ke- n , kecamatan ke- i , dan tahun ke- t .

ω_{in} = faktor penimbang sampel rumah tangga ke- n , kecamatan ke- i , tahun ke- t .

N_{it} = jumlah rumah tangga di kecamatan ke- i tahun ke- t .

m = jumlah kecamatan.

T = tahun survei.

dengan kovarian matriks estimasi langsung:

$$\text{cov}(\hat{y}_{it_1}, \hat{y}_{it_2}) = \frac{1}{(\hat{N}_{it}^{dir})^2} \sum_{n \in \omega_{it}} \omega_{in} (\omega_{in} - 1) (y_{it_1n} - \hat{y}_{it_1}) (y_{it_2n} - \hat{y}_{it_2}).$$

Penerapan Model

Identifikasi Model Persamaan Simultan

Model SERY:

$$\begin{array}{l} Y_1 = f(X_1, X_2, X_3) \\ Y_2 = f(X_1, X_4, X_5) \end{array} \longrightarrow \begin{array}{l} Y_1 = f(Y_2, X_1, X_2, X_3) \\ Y_2 = f(Y_1, X_1, X_4, X_5) \end{array}$$

Table 1

The order condition of identifiability of the simultaneous equation models.

Equations	$(q - q_d)$	$(D_d - 1)$	Decision	Results
Y_1	$5 - 3 = 2$	$2 - 1 = 1$	$(q - q_d) > (D_d - 1)$	Overidentified
Y_2	$5 - 3 = 2$	$2 - 1 = 1$	$(q - q_d) > (D_d - 1)$	Overidentified

Table 2

The rank conditions of identifiability of the simultaneous equation models.

Equations	Rank	Results
Y_1	$Rank(\Delta_{Y_2}) \neq 0$	Eligible rank condition
Y_2	$Rank(\Delta_{Y_1}) \neq 0$	Eligible rank condition

Table 3

Hausman test results.

Equations	F-Statistics	Variables	t-Statistics	Results
Y_1	13.0764*	Residual Y_2	7.3016*	Simultaneous
Y_2	12.2814*	Residual Y_1	7.2764*	Simultaneous

* Significant at $\alpha = 5 \%$.

Penerapan Model



Hasil Estimasi Parameter Model SERY

Table 4
Regression parameter estimates

Response	Variables	Coefficient	Std. error	p-value	Lower bound	Upper bound
Y_1	Y_2	0.14349	0.06539	0.02820 *	0.01533	0.27165
	Intercept	10.04024	0.83027	0.00000 *	8.41291	11.66757
	$\log X_1$	0.12348	0.00690	0.00000 *	0.10996	0.13700
	$\log X_2$	-0.00836	0.00158	0.00000 *	-0.01146	-0.00526
	$\log X_3$	-0.01496	0.00523	0.00425 *	-0.02521	-0.00471
Y_2	Y_1	0.79020	0.16452	0.00000 *	0.46774	1.11266
	Intercept	4.06886	2.04327	0.04644 *	0.06405	8.07367
	$\log X_1$	-0.17468	0.02667	0.00000 *	-0.22695	-0.12241
	X_4	-0.30562	0.12510	0.01456 *	-0.55082	-0.06042
	X_5	2.32869	0.49151	0.00000 *	1.36533	3.29205

*Significant at $\alpha = 5 \%$.

Table 5
Variance component parameter estimates.

Response	Component	Estimate
Y_1	$\hat{\sigma}_v^2$	0.009128
	$\hat{\sigma}_e^2$	0.053302
	$\hat{\rho}$	0.329574
Y_2	$\hat{\sigma}_v^2$	0.009175
	$\hat{\sigma}_e^2$	0.053501
	$\hat{\rho}$	0.102957

Penerapan Model



Hasil Estimasi Parameter Model SERY

Table 6

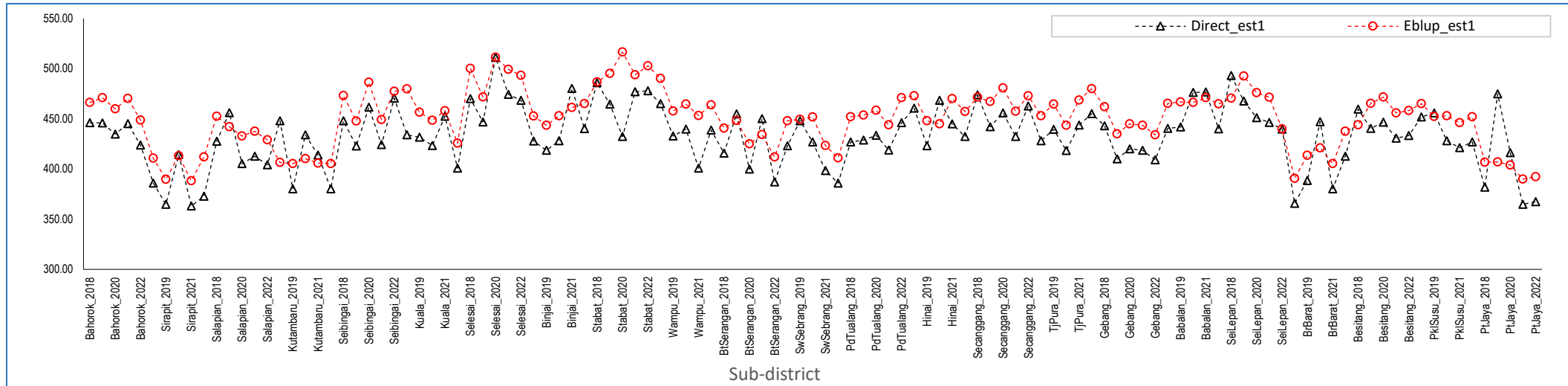
Summary of the CPE estimator

The EBLUP of SERY model in Langkat Regency, Indonesia, 2018–2022.

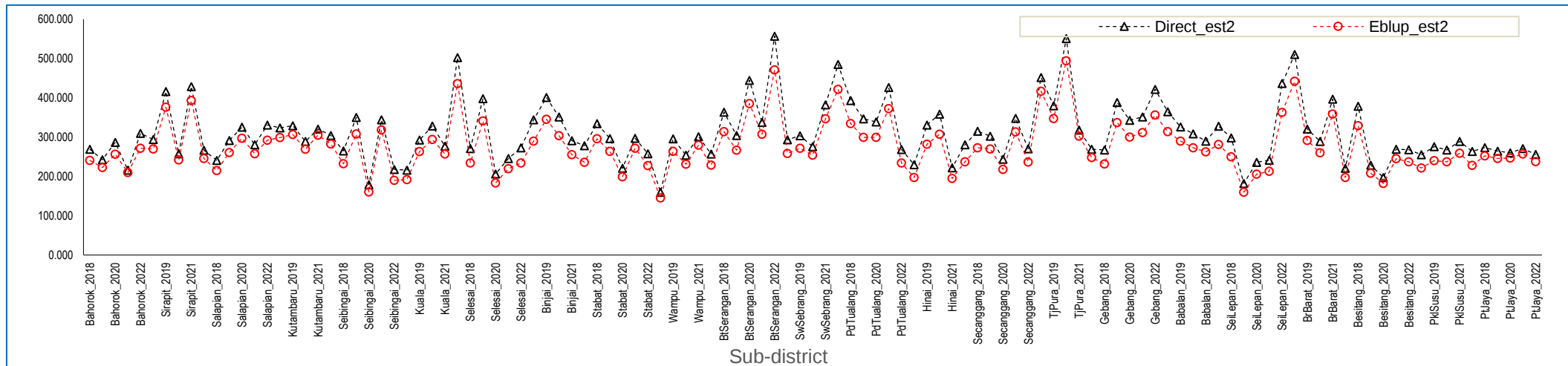
Components	2018	2019	2020	2021	2022
CPE of food (IDR)					
Minimum	395,905.17	390,323.05	386,126.38	380,370.10	376,773.69
Maximum	502,569.89	501,570.77	514,571.61	490,784.30	491,088.68
Median	453,504.29	451,215.35	450,913.14	449,503.99	451,626.14
Average	454,373.08	450,877.17	445,543.55	448,007.70	449,934.48
Percentage	65.56	65.10	66.23	65.36	64.73
Q1	436,306.39	430,778.79	419,983.20	432,047.15	435,121.25
Q2	453,504.29	451,215.35	450,913.14	449,503.99	451,626.14
Q3	474,738.78	469,465.46	465,003.40	466,297.91	474,207.37
Quartile deviation	19,216.20	19,343.34	22,510.10	17,125.38	19,543.06
Standard deviation	29,024.34	26,388.38	29,156.21	24,693.81	30,943.22
Range	106,664.72	111,247.72	128,445.23	110,414.20	114,314.99
Range deviation	26,666.18	27,811.93	32,111.31	27,603.55	28,578.75
CPE of non-food (IDR)					
Minimum	132,902.36	143,599.00	137,127.99	168,692.03	165,601.57
Maximum	383,996.99	326,681.18	410,158.93	335,662.01	402,282.56
Median	234,786.06	239,560.19	224,434.68	232,857.04	217,081.90
Average	238,641.92	241,677.63	227,158.35	237,407.26	245,192.70
Percentage	34.44	34.90	33.77	34.64	35.27
Q1	201,745.04	223,342.11	183,494.50	214,312.39	207,343.06
Q2	234,786.06	239,560.19	224,434.68	232,857.04	217,081.90
Q3	268,968.62	270,914.89	251,329.47	256,216.99	259,079.44
Quartile deviation	33,611.79	23,786.39	33,917.49	20,952.30	25,868.19
Standard deviation	55,633.48	39,672.33	57,814.01	43,761.06	64,389.77
Range	251,094.63	183,082.18	273,030.94	166,969.98	236,680.99
Range deviation	62,773.66	45,770.55	68,257.74	41,742.50	59,170.25

Penerapan Model

Hasil
Estimasi



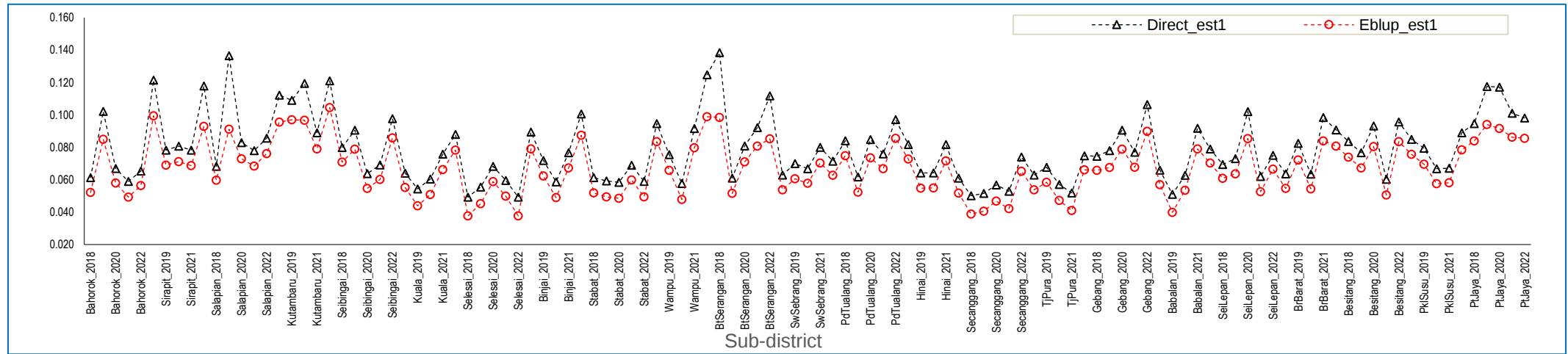
Gambar 6.1 Pengeluaran per kapita makanan Hasil Estimasi Langsung dan Model SERY



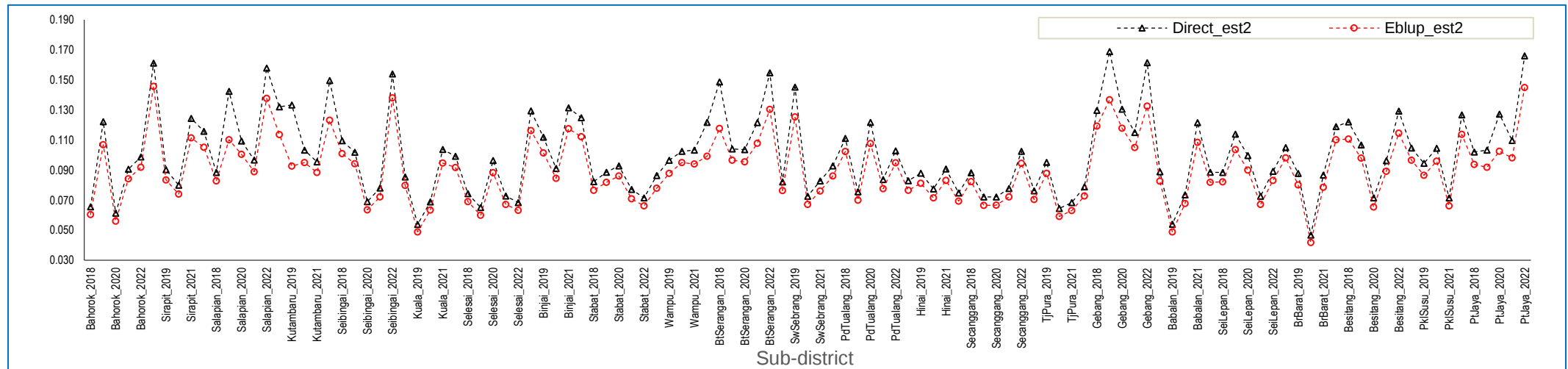
Gambar 6.2 Pengeluaran per kapita non-makanan Hasil Estimasi Langsung dan Model SERY

Penerapan Model

➔ RMSE



Gambar 6.3 RMSE Pengeluaran per kapita Makanan Hasil Estimasi Langsung dan Model SERY



Gambar 6.4 RMSE Pengeluaran per kapita Non-makanan Hasil Estimasi Langsung dan Model SERY

KESIMPULAN



Kesimpulan

Kesimpulan

01

Berdasarkan metode 3SLS-REML, penelitian ini telah berhasil mendapatkan estimator EBLUP model SERY. Untuk mengukur efisiensi estimasi dari model yang dikembangkan, penelitian ini juga telah mendapatkan estimator *Mean Square Error* (MSE) model SERY.

Estimator EBLUP :

$$\hat{\theta}_{SERY} = \mathbf{X}_A \hat{\beta}_{A_{3SLS-REML}} + \mathbf{ZGZ}'\Omega^{-1}[\mathbf{y} - \mathbf{X}_A \hat{\beta}_{A_{3SLS-REML}}]$$

dengan

$$\hat{\beta}_{A_{3SLS-REML}} = (\hat{\mathbf{X}}_A' \Omega^{-1} \hat{\mathbf{X}}_A)^{-1} \hat{\mathbf{X}}_A' \Omega^{-1} \mathbf{y},$$

$$\Omega = \mathbf{I}_D \otimes (\mathbf{I}_m \otimes [\sigma_v^2 \mathbf{J}_T + \sigma_\varepsilon^2 \mathbf{\Pi} + \sigma_\eta^2 \mathbf{I}_T]).$$

02

Estimator MSE dari model SERY diperoleh dengan mengambil elemen diagonal matriks kovarians EBLUP. Estimator MSE :

$$mse(\hat{\theta}) \approx \mathbf{g}_1(\hat{\delta}) + \mathbf{g}_2(\hat{\delta}) + 2\mathbf{g}_3(\hat{\delta}).$$

Kesimpulan

dimana $\mathbf{g}_b(\delta) = \text{col}_{1 \leq h \leq DmT} [\Theta_b(\delta)]_h$, $h = 1, \dots, DmT$, $b = 1, 2, 3$ dan $[\Theta_b(\delta)]_h$ adalah elemen diagonal ke- h dari matriks $\Theta_b(\delta)$ dengan

$$\Theta_1(\delta) = \Gamma \mathbf{R},$$

$$\Theta_2(\delta) = (\mathbf{I} - \Gamma) \mathbf{X}_A (\mathbf{X}_A' \boldsymbol{\Omega}^{-1} \mathbf{X}_A)^{-1} \mathbf{X}_A' (\mathbf{I} - \Gamma)',$$

$$\Theta_3(\delta) \approx \sum_{j=1}^r \sum_{l=1}^r \text{cov}(\hat{\delta}_j, \hat{\delta}_l) \Gamma_{(j)} \boldsymbol{\Omega} \Gamma_{(l)}', \quad j, l = 1, \dots, r$$

03

Hasil simulasi estimasi parameter model SERY dari data yang dibangkitkan sebanyak 100 kali pengulangan menunjukkan bahwa rata-rata estimasi parameter sangat dekat dengan nilai parameter populasi atau distribusi selisih antara estimasi dan parameter populasi berada di sekitar nol. Hal yang sama dilakukan untuk mengevaluasi parameter model SERY. Berdasarkan beberapa skenario model, penambahan jumlah area m dan series waktu t akan menurunkan bias serta meningkatkan efisiensi estimasi parameter.

Kesimpulan

04 Penerapan pada data riil diperoleh bahwa variabel pengeluaran per kapita non-makanan, jumlah penduduk, luas panen produksi tanaman pangan, dan jumlah industri mempunyai pengaruh signifikan terhadap estimasi pengeluaran per kapita makanan. Variabel pengeluaran per kapita makanan, jumlah penduduk, rasio fasilitas kesehatan, dan rasio fasilitas pendidikan mempunyai pengaruh signifikan terhadap estimasi pengeluaran per kapita non-makanan.

05 Rata-rata RMSE estimasi pengeluaran per kapita makanan dari seluruh kecamatan selama lima tahun periode survei berdasarkan estimasi langsung adalah 0,07897 dan berdasarkan estimasi model SERY sebesar 0,06720. Rata-rata RMSE estimasi pengeluaran per kapita non-makanan berdasarkan estimasi langsung adalah 0,09516 dan berdasarkan model SERY sebesar 0,09012. Kesimpulannya adalah estimasi berdasarkan model SERY memberikan hasil yang lebih baik dibanding estimasi langsung karena menghasilkan nilai RMSE yang lebih kecil. Hal ini disebabkan karena estimasi dengan model SERY mampu mengakomodir keragaman antar area dan waktu selama periode 2018-2022.

SARAN



Saran



Saran

Kepada BPS

- 01** Data hasil survei yang dipublikasikan oleh BPS umumnya berdasarkan hasil survei di tahun yang bersangkutan, dimana estimasinya didasarkan pada data cross section. Berdasarkan penelitian ini, penggunaan data panel meningkatkan akurasi dalam estimasi, sehingga disarankan untuk estimasi data hasil survei bisa mempertimbangkan penggunaan data series.
- 02** BPS bisa melakukan estimasi parameter pada area yang lebih kecil (kecamatan/desa/kelurahan) dengan menggunakan metode SAE. Model SERY bisa menjadi alternatif untuk estimasi pada area kecil dengan menggunakan data series.
- 03** Untuk memperbaiki kualitas estimasi parameter yang akan dipublikasikan oleh BPS khususnya di area kecil maka dibutuhkan ketersediaan informasi/data yang konsisten antar area dan antar waktu sedemikian rupa sehingga dapat dijamin bahwa kualitas estimasinya menjadi lebih baik dibandingkan dengan estimasi langsung.

Saran



Saran

Kepada Peneliti

- 01** Keterbatasan model yang dikembangkan dalam penelitian ini adalah mengasumsikan pengaruh acak area adalah stasioner, sehingga tidak dapat menangkap variasi atau disparitas antar area. Hal ini memberikan tantangan bagi penelitian selanjutnya, untuk mengeksplorasi persamaan simultan untuk estimasi area kecil dengan model dinamis dengan memodifikasi pengaruh acak area dan menghilangkan batasan stasioneritas, sehingga mampu menangkap kondisi dimana kesenjangan antar area semakin berkurang dari waktu ke waktu..
- 02** Metode yang dikembangkan dalam penelitian ini didasarkan pada asumsi normalitas untuk pengaruh acak dan error. Pada praktiknya, asumsi normalitas bisa saja tidak terpenuhi karena adanya data khusus seperti potensi outlier. Oleh karena itu, untuk penelitian selanjutnya bisa mengembangkan model robust Simultaneous Equation Rao-Yu untuk mengatasi keberadaan outlier.
- 03** Model SERY merupakan model campuran linier yang dirancang untuk variabel kontinu. Oleh karena itu, untuk penelitian selanjutnya bisa mengembangkan model SERY untuk data biner atau kategorik dengan pendekatan model persamaan nonlinier simultan atau bayesian.
- 04** Model SERY yang dikembangkan dalam penelitian ini hanya bisa digunakan untuk memodelkan area tersurvei. Oleh karena itu, untuk penelitian selanjutnya bisa mengembangkan model SERY untuk area tidak tersurvei..



DAFTAR PUSTAKA



Daftar Pustaka

- Albacea, Z.V.J. (2003). Estimating Sub-national Poverty Incidence Using Small Area Estimation Technique. RETA 6007: Enhancing Social and Gender Statistics, Bangkok.
- Anderson, T. W., & Rubin, H. (1949). Estimation of the Parameters of a Single Equation in a Complete System of Stochastic Equations. *The Annals of Mathematical Statistics*, 20(1), 46–63.
- Baltagi, Badi H. (2005). *Econometric Analysis of Panel Data*. New York: John Wiley & Sons.
- Benavent, R., & Morales, D. (2016). Multivariate Fay-Herriot models for small area estimation. *Computational Statistics and Data Analysis*, 94, 372–390. <https://doi.org/10.1016/j.csda.2015.07.013>
- Best, N., Richardson, S., & Clarke, P. (2008). A Comparison of model-based methods for Small Area Estimation. *ESRC National Centre for Research Methods*, 1–26.
- BPS. (2019). *Data dan Informasi Kemiskinan Kabupaten/Kota Tahun 2019*. Jakarta: Badan Pusat Statistik.
- Brundy, J. M., & Jorgenson, D. W. (1971). Efficient Estimation of Simultaneous Equations by Instrumental Variables. *The Review of Economics and Statistics*, 53(3), 207-224.
- Catalán, H. E. N., Fifita, V. K., & Faingaanuku, W. (2020). Small-Area Multidimensional Poverty Estimates for Tonga 2016: Drawn from a Hierarchical Bayesian Estimator. *Applied Spatial Analysis and Policy*, 13(2), 305–328. <https://doi.org/10.1007/s12061-019-09304-8>
- Datta, G. S., & Lahiri, P. (2000). A unified measure of uncertainty of estimated best linear unbiased predictors in small area estimation problems. *Statistica Sinica*, 10(2), 613–627.
- Datta, G. S., Lahiri, P., Maiti, T., & Lu, K. L. (1999). Hierarchical Bayes Estimation of Unemployment Rates for the States of the U.S. *Journal of the American Statistical Association*, 94(448), 1074–1082.
- Datta, G. S., Lahiri, P., & Maiti, T. (2002). Empirical Bayes Estimation of Median Income of Four Person Families by State Using Time Series and Cross Sectional Data. *Journal of Statistical Planning and Inferences*. 102: 83-97
- Dhrymes, P. J. (1973). Small Sample and Asymptotic Relations Between Maximum Likelihood and Three Stage Least Squares Estimators. *Econometrica*, 41(2), 357. doi:10.2307/1913495

Daftar Pustaka

- Esteban, M. D., Morales, D., Pérez, A., & Santamaría, L. (2012). Small area estimation of poverty proportions under area-level time models. *Computational Statistics and Data Analysis*, 56(10), 2840–2855. <https://doi.org/10.1016/j.csda.2011.10.015>
- Fay, R.E., & Herriot, R.A. (1979). Estimates of income for small places: an application of James-Stein procedures to census data. *Journal of the American Statistical Association* 74. 269- 277.
- Fay, R.E. (1987). Application of multivariate regression of small domain estimation, In: Platek R, Rao JNK, Saˆrndal CE, Singh MP, Eds. *Small Area Statistics*, John Wiley & Son. Inc, New York, 91-102.
- Fay, R. E., Planty, M., & Diallo, M. S. (2013). Small Area Estimation Alternatives for the National Crime Victimization Survey. *Proceedings of the Jont Statistical Meetings*. 1544–1557.
- Ghosh, M., & Rao, J. N. K. (1994). Small area estimation: An appraisal. *Statistical Science*, 9(1), 55–76. <https://doi.org/10.1214/ss/1177010647>
- Greene, W.H. (2003). *Econometrics Analysis*. 5th Ed. New Jersey: Prentice Hall.
- Gujarati, D. (2004). *Basic Econometric: Fourth Edition*. Mc.Graw Hill Companies.
- Hansen, L. P., Heaton, J., & Yaron, A. (1996). Finite-sample properties of some alternative GMM estimators. *Journal of Business and Economic Statistics*, 14(3), 262–280.
- Harville, D.A. (1991). That BLUP is a good thing: the estimation of random effects. Discussion on Robinson paper. *Statistical Science*, 6, 15-51.
- Henderson, C. R. (1953). Estimation of Variance and Covariance Components. *Biometrics*, 9(2), 226. doi:10.2307/3001853
- Henderson, C. R. (1975). Best Linear Unbiased Estimation and Prediction Under a Selection Model. *Biometrics* 31, 423-447.
- Hidiroglou, M. (2007). Small-Area Estimation: Theory and Practice. In *Proceedings of the Survey Research Methods Section*, (pp. 3445-3456).
- Hsiao, C. (2003). *Analysis of Panael Data*. New York : Cambridge University Press.
- Jedrzejczak, A., & Kubacki, J. (2017). Estimation of Small Area Characteristics Using Multivariate Rao-Yu Model. *Statistics and Transition new series*, Vil. 18, No 4, pp. 725-742. DOI: 10.21307/stattrans-2017-009.
- Kackar, R.N, & Harville, D.A. (1984). Approximations for standard errors of estimators of fixed and random effect in mixed linear models. *J Amer Stat Ass.* 79:853-862.

Daftar Pustaka

- Kurnia, A., & Notodiputro, K. A. (2006). EB-EBLUP MSE Estimator on Small Area Estimation with Application to BPS Data. Proceeding at The First International Conference on Mathematics and Statistics (IcoMS-1), June 19-21, Bandung
- Lahiri, P., & Rao, J. N. K. (1995). Robust estimation of mean squared error of small area estimators. *Journal of the American Statistical Association*, 90(430), 758–766. <https://doi.org/10.1080/01621459.1995.10476570>
- Longford, N.T. (2005). Missing Data and Small-Area Estimation: Modern Analytical Equipment for the Survey Statistician. In *Technometrics* (Vol. 49, Issue 3). <https://doi.org/10.1198/tech.2007.s520>
- Marhuenda, Y., Molina, I., & Morales, D. (2013). Small area estimation with spatio-temporal Fay-Herriot models. *Computational Statistics and Data Analysis*, 58(1), 308–325. <https://doi.org/10.1016/j.csda.2012.09.002>
- Molina, I., Nandram, B., & Rao, J. N. K. (2014). Small area estimation of general parameters with application to poverty indicators: A hierarchical bayes approach. *Annals of Applied Statistics*, 8(2), 852–885.
- Molina, I., Rao, J. N. K., & Datta, G. S. (2015). Small area estimation under a Fay–Herriot model with preliminary testing for the presence of random area effects. *Survey Methodology*, 41(1), 1- 19.
- Molina, I., & Rao, J. N.K. (2010). Small Area Estimation of Poverty Indicators. Working Paper 09-15 Statistics and Econometrics Series 05, March 2009.
- Muchlisoh, S. (2017). Pengembangan Model Pendugaan Area Kecil dengan Pengaruh Acak Waktu Mengikuti Proses Autoregresif Ordo Pertama. [Disertasi]. Bogor (ID): Institut Pertanian Bogor.
- Noviyanti, R.A., Setiawan, Rumiati, A.T. (2024). Small area estimation of consumption per capita expenditure under simultaneous equation Rao-Yu model. *MethodsX*, 14C (2025), 103083.
- Patterson, H. D., & Thompson, R. (1971). Recovery of inter-block information when block sizes are unequal. *Biometrika*, 58(3), 545–554. <https://doi.org/10.1093/biomet/58.3.545>
- Pfeiffermann, D., Terryn, B., & Moura, F. A. S. (2008). Small area estimation under a two-part random effects model with application to estimation of literacy in developing countries. *Survey Methodology*, 34(2), 235–249.
- Prasad, N. G. N., & Rao, J. N. K. (1990). The estimation of the mean squared error of small-area estimators. *J Amer Stat Ass*, 85:163-171.

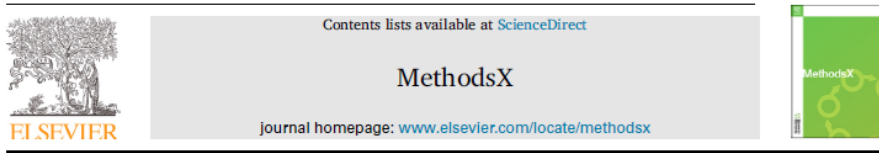
Daftar Pustaka

- Pratesi, M., & Salvati, N. (2008). Small area estimation: The EBLUP estimator based on spatially correlated random area effects. *Statistical Methods and Applications*, 17(1), 113–141. <https://doi.org/10.1007/s10260-007-0061-9>
- Puspongoro, N. H., & Rachmawati, R. N. (2018). Spatial Empirical Best Linear Unbiased Prediction in Small Area Estimation of Poverty. *Procedia Computer Science*, 135, 712–718. <https://doi.org/10.1016/j.procs.2018.08.214>
- Rao, J.N.K. (2003). *Small Area Estimation*. New York: John Wiley and Sons, Inc.
- Rao, J.N.K., & Molina, I. (2015). *Small Area Estimation*, second edition. New York: John Wiley and Sons.
- Rao, J. N. K., & Yu, M. (1992). Small Area Estimation Combining Time Series and Cross Sectional Data. *Proceedings of the Joint Statistical Meetings*. 1-9.
- Rao, J. N. K., & Yu, M. (1994). Small-area estimation by combining time-series and cross-sectional data. *Canadian Journal of Statistics*, 22(4), 511–528. <https://doi.org/10.2307/3315407>
- Robinson, G. K. (1991). That BLUP is a Good Thing: The Estimation of Random Effects]: Rejoinder. *Statistical Science*, 6(1), 48–51. <https://doi.org/10.1214/ss/1177011933>
- Rumiati, A.T. (2012). *Model Bayes Untuk Pendugaan Area Kecil dengan Penarikan Contoh Berpeluang Tidak Sama pada Kasus Respon Binomial dan Multinomial*. [disertasi]. Bogor (ID): Institut Pertanian Bogor.
- Sadik, K., & Notodiputro, K. A. (2006). Metode E-Blup Dalam Small Area Estimation Untuk Model Yang Mengandung Random Walk. *Forum Statistika Dan Komputasi*, 11(2).
- Sadik, K., & Notodiputro, K. A. (2008). Small Area Estimation With Time And Area Effects Using A Dynamic Linear Model. *Proceeding The 3th International Conference on Mathematics and Statstics (ICoMS)*. Bogor
- Sainath, J. (2014). *Structural Equation Mixed Models with an Application to Small Area Estimation*. [Dissertation], Lincoln, Nebraska.
- Sargan, J. D. (1958).. On the Estimation of Economic Relationships by Means of Instrumental Variables. *Econometrica*, 26:393-415.
- Susianto, Y. (2017). *Modifikasi Model Rao-Yu untuk Pendugaan Area Kecil Musiman dengan Penerapan Pada Data Susenas* [disertasi]. Bogor (ID): Institut Pertanian Bogor.

Daftar Pustaka

- Suwarti, et al. (2019). Spatio Temporal Fay Herriot Modelsin Small Area Estimation to Obtain Factors that Affecting Poverty in Polewali Mandar District. IOP Publising. 546 (2019) 052076. doi:10.1088/1757-899X/546/5/052076
- Theil, H. (1961). Economics Forecasts and Policy, 2nd ed. North-Holland, Amsterdam.
- Todaro, M. P., & Smith, S. C. (2015). Economic development: The Addison-Wesley series in economics.
- Ubaidillah, A. (2014). Small Area Estimation dengan Pendekatan Hierarchical Bayesian Neural Network Untuk Pemetaan Kemiskinan di Kota Jambi. [Thesis]. Surabaya (ID): ITS.
- Ubaidillah, A. (2017). Simultaneous Eqution Models for Small Area Estimation. [Disertasi]. Bogor (ID): Institut Pertanian Bogor.
- Ubaidillah, A., Notodiputro, K. A., Kurnia, A., & Mangku, I. W. (2019). Multivariate Fay-Herriot models for small area estimation with application to household consumption per capita expenditure in Indonesia. Journal of Applied Statistics, 46(15), 2845–2861. <https://doi.org/10.1080/02664763.2019.1615420>
- Wooldridge, J. M. (2010). Econometric analysis of cross-section and panel data. Cambrigde: MIT press.
- Zellner, A., & Theil, H. (1962). Three Stage Least Squares: Simultaneous Estimation of Simultaneous Equations. Econometrica, 30(1), 54. doi:10.2307/1911287.

Publikasi



Small area estimation of consumption per capita expenditure under simultaneous equation Rao-Yu model ^{☆,☆☆}

Reny Ari Noviyanti ^{a,b}, Setiawan ^{a,*}, Agnes Tuti Rumiati ^a

^aDepartment of Statistics, Faculty of Science and Data Analytics, Institut Teknologi Sepuluh Nopember, Surabaya 60111, Indonesia
^bBPS, Statistics of Sumatera Utara Province, Medan 20123, Indonesia

ARTICLE INFO

Method name:
Simultaneous Equation Rao-Yu (SERV) model

Keywords:
3SLS
EBLUP
CPE
MSE
Rao-Yu model
REML
SAE
Simultaneous equation

ABSTRACT

This article proposed a simultaneous equation model for small area estimation with random area and time-varying effects called the Simultaneous Equation Rao-Yu (SERV) model. In the context of small area estimation, many socioeconomic variables are likely to exhibit not only correlations but also causal relationships. Therefore, it is considered to use simultaneous equation model for indirect estimation method in a small-area. The SERV model was developed to accommodate causal relationships among the variables of interest in time series and cross-sectional data. The SERV model is a modification of the Rao-Yu model, which was constructed using simultaneous equation that allows endogenous variables as explanatory variables. For fitting linear mixed models, three-stage least squares restricted maximum likelihood method was proposed to derive the empirical best linear unbiased predictor and mean squared error estimator. Finally, the model was applied to estimate consumption per capita expenditure of food and non-food.

<https://doi.org/10.1016/j.mex.2024.103083>

AIP Conference Proceedings

RESEARCH ARTICLE | NOVEMBER 15 2024

Empirical best linear unbiased predictions of small area estimation with panel data for estimating household consumption per capita expenditure ^{FREE}

Reny Ari Noviyanti; Setiawan [✉]; Agnes Tuti Rumiati
AIP Conf. Proc. 3201, 060005 (2024)
<https://doi.org/10.1063/5.0231010>



Articles You May Be Interested In

Small area estimation of poverty severity index for Kecamatan (sub-district) in Surabaya city in 2020 using empirical best linear unbiased prediction method

AIP Conf. Proc. (April 2024)

Relationship model among human development index, Gini coefficient, per capita non-food expenditure, per capita gross regional domestic product and development financing in provinces in Indonesia for the 2015-2019 period

AIP Conf. Proc. (December 2023)

The effectiveness of small area estimation with best linear unbiased prediction method on various sample sizes with resampling simulation on SUSENAS data in the case of mean years school of Kecamatan (sub-district) in Surabaya

AIP Conf. Proc. (April 2024)

<https://doi.org/10.1063/5.0231010>



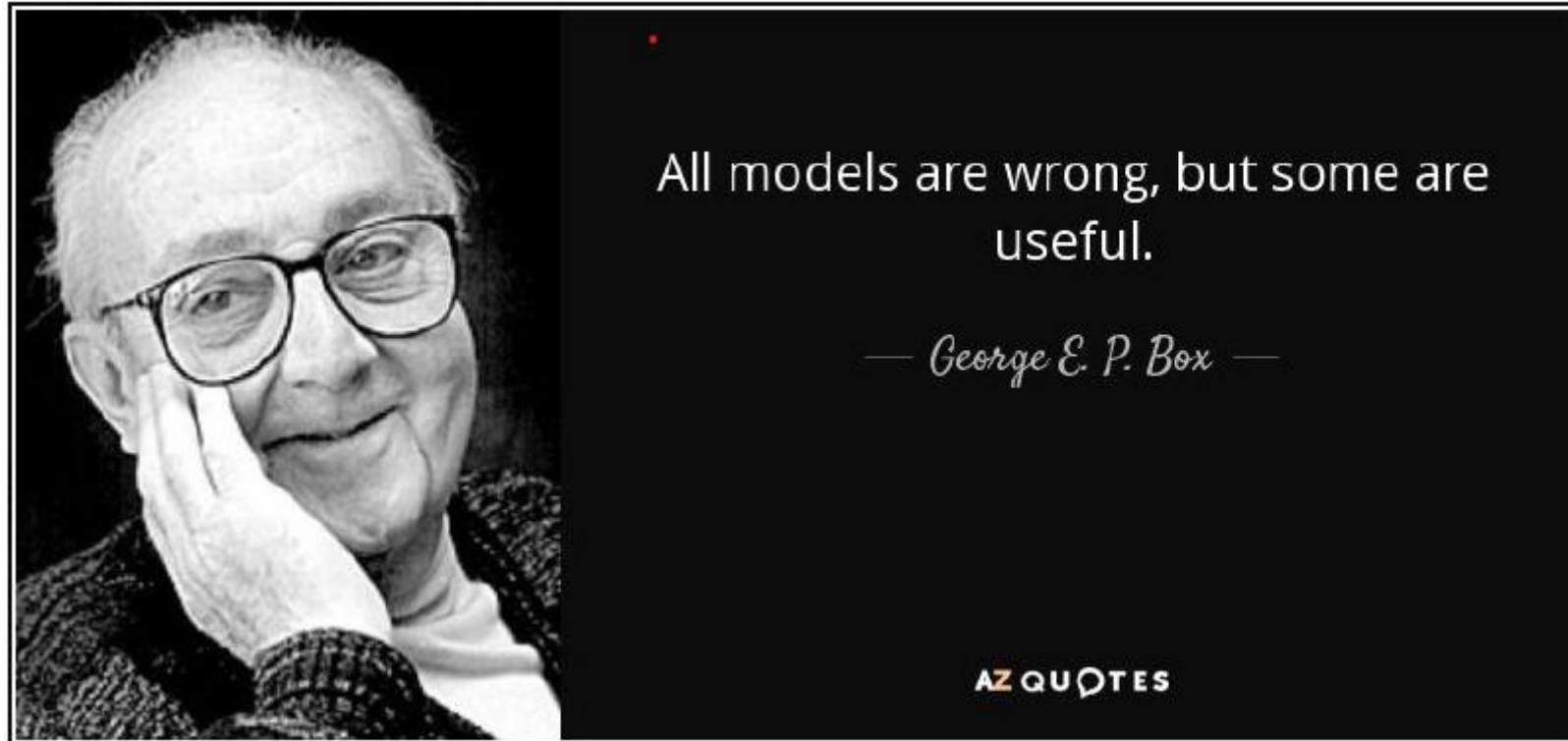
Small Area Estimation of Mean Years of Schooling Under Time Series and Cross-sectional Models

Reny Ari Noviyanti, Setiawan, and Agnes Tuti Rumiati

Abstract Small area estimation develops within the framework of time series and cross-sectional models. The restricted estimation maximum likelihood method was used to obtain the empirical best linear unbiased prediction for small areas and its mean squared error estimators. The model focuses on applying statistical models that permit borrowing strength over area and time. The process uses regularly conducted survey data, where the areas of interest are observed repeatedly under a predetermined scheme. The time series and cross-sectional models were able to capture heterogeneity across area and time, so it can be used to enhance sample size effectiveness, thereby minimizing mean squared error and producing a more accurate estimation. The simulation results show that the degree of correlation parameters significantly affects the efficiency of the model. The application to estimate mean years of schooling at the sub-district level in Langkat Regency, North Sumatra, Indonesia, for the period of 2018–2021 showed that the time correlation coefficient was 0.3758, the variance of the area random effect was 1.1125, and the variance of the area-time random effect was 0.3241. The estimations derived from time series and cross-sectional models had a lower mean squared error than those obtained from the Fay-Herriot models and direct estimation.

https://doi.org/10.1007/978-981-97-0293-0_26

Penutup



TERIMA KASIH

